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Inventory model with stock-level dependent demand rate and variable holding cost

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Abstract

Inventory models in which the demand rate depends on the inventory level are based on the common real-life observation that greater product availability tends to stimulate more sales. Previous models incorporating inventory-level dependent demand rate assume that the holding cost is constant for the entire inventory cycle. This paper considers the inventory policy for an item with a stock-level dependent demand rate and a storage-time dependent holding cost. The holding cost per unit of the item per unit time is assumed to be an increasing function of the time spent in storage. Two time-dependent holding cost step functions are considered: Retroactive holding cost increase, and incremental holding cost increase. Procedures are developed for determining the optimal order quantity and the optimal cycle time for both cost structures.

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1. Introduction

In traditional inventory models, the demand rate is assumed to be a given constant. Various inventory models have been developed for dealing with varying and stochastic demand. All these models implicitly assume that the demand rate is independent, i.e. an external parameter not influenced by the internal inventory policy. In real life, however, it is frequently observed that demand for a particular product can indeed be influenced by internal factors such as price and availability. The change in the demand in response to inventory or marketing decisions is commonly referred to as demand elasticity.

Most models that consider demand variation in response to item availability (i.e. inventory level) assume that the holding cost is constant for the entire inventory cycle. This paper presents an inventory model with a stock-level dependent demand rate and a variable holding cost. In this model, the holding cost is an increasing step function of the time spent in storage. Two types of time-dependent holding cost increase functions are considered: Retroactive increase, and incremental increase. For each type, a simple algorithm that minimizes the total inventory cost (TIC) is developed for calculating the optimal order quantity and associated cycle time.

As far as the author knows, the step structure of the holding cost function is unique to this paper. This structure is representative of many real-life situations in which the storage times can be

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classified into different ranges, each with its distinctive unit holding cost. This is particularly true in the storage of deteriorating and perishable items such as food products. The longer these food products are kept in storage, the more sophisticated the storage facilities and services needed, and therefore, the higher the holding cost. For example, three different holding cost rates may apply to short-term, medium-term, and long-term food storage.

The remainder of this paper is organized as follows. Relevant literature is reviewed in the next section. This is followed by defining the problem and scope and developing the inventory model. Subsequently, the model is analyzed and a solution algorithm is presented for Case 1, retroactive holding cost increase. A similar analysis is followed for Case 2, incremental holding cost increase. Finally, suggestions and concluding remarks are given.

2. Literature review

Various models have been proposed for stock-level dependent inventory systems. Baker and Urban (1988a) investigated a deterministic inventory system in which the demand rate dependence on the inventory level is described by a polynomial function. A non-linear programming algorithm is utilized to determine the optimal order size and the reorder point. Urban (1995) investigated an inventory system in which the demand rate during stock-out periods differs from the in-stock period demand by a given amount. The demand rate depends on both the initial stock and the instantaneous stock. Urban formulates a profit-maximizing model and develops a closed-form solution.

A number of authors investigated inventory systems with a two-stage demand rate. Baker and Urban (1988b) considered an inventory system with an initial period of level-dependent demand followed by a period of constant demand. The analysis conducted on this model imposes a terminal condition of zero inventories at the end of the order cycle. Datta and Pal (1990) analyzed an infinite time horizon deterministic inventory system without shortage, which has a level-dependent demand rate up to a certain stock level and a constant demand for the rest of the cycle. Paul et al. (1996) investigated a deterministic inventory system in which shortages are allowed and are fully back-logged. The demand is stock dependent to a certain

level and then constant for the remaining periods. A flow chart is provided to solve the general system.

Pal et al. (1993) developed a deterministic inventory model assuming that the demand rate is stock dependent and that the items deteriorate at a constant rate θ. The net profit over one production run is maximized by numerically solving two nonlinear equations, and the optimal solution is compared with the no deterioration ($\theta = 0$) case. Hwang and Hahn (2000) constructed an inventory model for an item with an inventory-level dependent demand rate and a fixed expiry date. All units that are not sold by their expiry date are regarded as useless and therefore discarded. Separable programming is utilized to determine the optimal order level and order cycle length.

The holding cost is explicitly assumed to be varying over time in only few inventory models. Giri et al. (1996) developed a generalized EOQ model for deteriorating items with shortages, in which both the demand rate and the holding cost are continuous functions of time. The optimal inventory policy is derived assuming a finite planning horizon and constant replenishment cycles. Ray and Chaudhuri (1997) take the time value of money into account in analyzing an inventory system with stock-dependent demand rate and shortages. Two types of inflation rates are considered: internal (company) inflation, and external (general economy) inflation.

Shao et al. (2000) determined the optimum quality target for a manufacturing process where several grades of customer specifications may be sold. Since rejected goods could be stored and sold later to another customer, variable holding costs are considered in the model. Beltran and Krass (2002) analyzed the dynamic lot sizing problem with positive or negative demands and allowed disposal of excess inventory. Assuming deterministic time-varying demands and concave holding costs, an efficient dynamic programming algorithm is developed for this finite time horizon problem.

Goh (1994) apparently provides the only existing inventory model in which the demand is stock dependent and the holding cost is time dependent. Actually, Goh (1994) considers two types of holding cost variation: (a) a nonlinear function of storage time and (b) a nonlinear function of storage level. In this paper, we present a different functional form of the holding cost storage time dependence. While Goh (1992) models holding cost variation over time as a continuous nonlinear function, this paper

introduces two types of discontinuous step functions. The storage time is divided into a number of distinct periods with successively increasing holding costs. As the storage time extends to the next time period, the new holding cost can be applied either retroactively (to all storage periods), or incrementally (to the new period only).

3. Problem definition and scope

The main objective of this paper is to determine the optimum (i.e. minimum cost) inventory policy for an inventory system with inventory-level dependent demand rate and a time-dependent holding cost. Assuming the demand rate to be inventory-level dependent means the demand is higher for greater inventory levels. Assuming the holding cost per unit of the item per unit time to be time dependent means the unit holding cost is higher for longer storage periods. The model that will be developed for the inventory system is based on allowing unit holding cost values to vary across different storage period. Variable unit holding costs are considered in the model in determining the optimal inventory policy.

The holding cost per unit is assumed to increase only when the storage time exceeds specified discrete values. In other words, the holding cost per unit per unit time is an increasing step function of the storage time. Two types of holding cost step functions are considered: Retroactive increase, and incremental increase. In retroactive increase, the unit holding cost rate of the last storage period is applied to all storage periods. In incremental increase, the rate of each period, including the last period, is applied only to units stored in that particular period.

3.1. Notation

The following notation is adopted from Goh (1992) for the model under consideration:

$q(t)$ the quantity on-hand at time t
D constant (base) demand rate
n number of distinct time periods with different holding cost rates
t time from the start of the cycle at $t = 0$
t_i end time of period i, where $i = 1, 2, \ldots, n$, $t_0 = 0$, and $t_n = \infty$
k ordering cost per order
h_i holding cost of the item in period i

$h(t)$ holding cost of the item at time t, $h(t) = h_i$ if $t_{i-1} \leqslant t \leqslant t_i$
T cycle time
β demand parameter indicating elasticity in relation to the inventory level

3.2. Assumption and limitations

1. The demand rate R is an increasing function of the inventory level q.
2. The holding cost is varying as an increasing step function of time in storage.
3. Replenishments are instantaneous.
4. Shortages are not allowed.
5. A single item is considered.
6. The demand rate R dependence on the inventory level q is expressed as
$$R(q) = Dq^{\beta}, \quad D > 0, \quad 0 < \beta < 1, \quad q \geqslant 0. \tag{1}$$

3.3. The inventory model

The objective is to minimize the TIC per unit time, which includes two components: The ordering cost, and the holding cost. Since one order is made per cycle, the ordering cost per unit time is simply K/T. The total holding cost per cycle is obtained by integrating the product of holding cost $h(t)$ and inventory level $q(t)$ over the whole cycle.

$$\text{TIC} = \frac{k}{T} + \frac{1}{T} \int_0^T h(t) q(t) \, dt. \tag{2}$$

Since the demand rate is equal to the rate of inventory level decrease, we can describe inventory level q by the following differential equation:

$$\frac{dq(t)}{dt} = -D[q(t)]^{\beta}, \quad D > 0, \quad 0 \leqslant t \leqslant T, \quad 0 < \beta < 1. \tag{3}$$

The on-hand inventory level at time t, $q(t)$, can be evaluated by solving (3):

$$q^{-\beta} \, dq = -D \, dt,$$

by integrating both sides:

$$\int_0^t q^{-\beta} \, dq = \int_0^t -D \, dt, \quad \text{where } 0 \leqslant t \leqslant T,$$

$$\left. \frac{q^{1-\beta}}{(1-\beta)} \right|_0^t = -Dt,$$

$$q^{1-\beta}(t) = D(1-\beta)t + q^{1-\beta}(0).$$

However,

$$q^{1-\beta}(0) = Q^{1-\beta}.$$

Thus,

$$q^{1-\beta}(t) = -D(1-\beta)t + Q^{1-\beta},$$

$$q(t) = [-D(1-\beta)t + Q^{1-\beta}]^{1/(1-\beta)}. \tag{4}$$

The period T can be evaluated by substituting the inventory function $q(t)$ at T:

$$-q^{1-\beta}(T) = [-D(1-\beta)T + Q^{1-\beta}]^{1/(1-\beta)} = 0.$$

Hence,

$$T = \frac{Q^{1-\beta}}{D(1-\beta)}, \tag{5}$$

or

$$Q = [D(1-\beta)T]^{1/(1-\beta)}. \tag{6}$$

4. Case 1: Retroactive holding cost increase

As stated earlier, the holding cost is assumed to be an increasing step function of storage time, i.e. $h_1 < h_2 < \cdots < h_n$. In Case 1, a uniform holding cost that depends on the length of storage is used. Specifically, the holding cost of the last storage period applies retroactively to all previous periods. Thus, if the cycle ends in period e, $(t_{e-1} \leqslant T \leqslant t_e)$, then the holding cost rate h_e is applied to all periods $1, 2, \ldots, e$. In this case, the TIC per unit time can be expressed as

$$\text{TIC} = \frac{k}{T} + \frac{h_i}{T} \int_0^T q(t)\,\mathrm{d}t, \quad t_{i-1} \leqslant T \leqslant t_i. \tag{7}$$

Substituting the value of $q(t)$ from (4)

$$\text{TIC} = \frac{k}{T} + \frac{h_i}{T} \int_0^T \left[-D(1-\beta)t + Q^{1-\beta} \right]^{1/(1-\beta)}\,\mathrm{d}tT,$$

$$= \frac{k}{T} - \frac{h_i}{D(2-\beta)T} \left[-D(1-\beta)\Big|_0^T + Q^{1-\beta} \right]^{(2-\beta)/(1-\beta)}.$$

Thus,

$$\text{TIC} = \frac{k}{T} + \frac{h_i}{D(2-\beta)T}$$
$$\times \left[[Q^{1-\beta}]^{(2-\beta)/(1-\beta)} - [-D(1-\beta)T + Q^{1-\beta}]^{(2-\beta)/(1-\beta)} \right].$$

Substituting the value of T from (5)

$$\text{TIC} = \frac{kD(1-\beta)}{Q^{1-\beta}} + \frac{h_i(1-\beta)Q}{(2-\beta)}, \quad t_{i-1} \leqslant T \leqslant t_i. \tag{8}$$

Setting the derivative of TIC with respect to Q equal to zero and solving for Q, we obtain:

$$Q^* = \left[\frac{kD(1-\beta)(2-\beta)}{h_i} \right]^{1/(2-\beta)}, \quad Q^* = t_{i-1} \leqslant T \leqslant t_i. \tag{9}$$

4.1. Solution algorithm

The optimum solution can be determined by using the following steps:

1. Starting with the lowest holding cost h_1, use (9) to determine Q and (5) to determine T for each h_i until Q is realizable (i.e. $t_{i-1} \leqslant T \leqslant t_i$). Call these values T_R and Q_R.
2. Use (6) to calculate all break-point values of Q, $Q_i = Q(t_i)$, $t_1 \leqslant t_i < T_R$; each Q_i is obtained by substituting t_i into (6).
3. Use (8) to calculate the TIC for Q_R and each Q_i.
4. Choose the value of Q that gives the lowest TIC.

4.2. Example 1

Given the following parameters.

$D = 400$ units per year,
$k = \$300$ per order,
$\beta = 0.1$,
$h_1 = \$5$/unit/year, $0 < T \leqslant 0.2$, $t_1 = 0.2$ year,
$h_2 = \$6$/unit/year, $0.2 < T \leqslant 0.4$, $t_2 = 0.4$ year,
$h_3 = \$7$/unit/year, $0.4 < T \leqslant \infty$, $t_3 = \infty$.

Step 1: Starting with $h_1 = 5$,

$$Q^* = \left[\frac{300(400)(1-0.1)(2-0.1)}{5} \right]^{1/1.9} = 268 \text{ units},$$

$$T = \frac{268^{0.9}}{400(1-0.1)} = 0.426 \text{ year} \quad \text{(not realizable)}.$$

Substituting $h_2 = 6$,

$$Q^* = \left[\frac{300(400)(1-0.1)(2-0.1)}{6} \right]^{1/1.9} = 243 \text{ units},$$

$$T = \frac{243^{0.9}}{400(1-0.1)} = 0.39 \text{ year} \quad \text{(realizable, } Q_R = 243\text{)}.$$

Step 2:

$$Q_1 = [(400(1-0.1)0.2]^{1/0.9} = 116 \text{ units},$$

$$Q_2 = [(400(1-0.1)0.4]^{1/0.9} = 250 \text{ units}.$$

Step 3:

$$\text{TIC}(243) = \frac{300(400)(1 - 0.1)}{243^{0.9}} + \frac{6(1 - 0.1)243}{(2 - 0.1)}$$
$$= 1460.43,$$

$$\text{TIC}(116) = \frac{300(400)(1 - 0.1)}{116^{0.9}} + \frac{5(1 - 0.1)116}{(2 - 0.1)}$$
$$= 1772.39.$$

Step 4: The optimum solution is:

$Q^* = 243$ units,

$T^* = 0.39$ year,

$TIC^* = \$1460.43/\text{year}$.

5. Case 2: Incremental holding cost increase

The holding cost is now assumed to be an incremental step function of storage time. According to this function, higher storage cost rates apply to storage in later periods. Thus, if the cycle ends in period e ($t_{e-1} \leqslant T \leqslant t_e$), then holding cost rate h_1 is applied to period 1, rate h_2 is applied to period 2, and so on; thus rate h_e is applied only to period e from time t_{e-1} up to time T. For this case, we first reset the value of t_e as ($t_e = T$), and then express the TIC per unit time as

$$\text{TIC} = \frac{k}{T} + \frac{h_1}{T} \int_0^{t_1} q(t)\,\mathrm{d}t + \frac{h_2}{T} \int_{t_1}^{t_2} q(t)\,\mathrm{d}t$$
$$+ \cdots + \frac{h_e}{T} \int_{t_{e-1}}^{t_e=T} q(t)\,\mathrm{d}t. \tag{10}$$

Substituting the value of $q(t)$ from (4), we obtain:

$$\text{TIC} = \frac{k}{T} + \sum_{i=1}^{e} \frac{h_i}{T} \int_{t_{i-1}}^{t_i} \left[-D(1 - \beta)t + Q^{1-\beta} \right]^{1/(1-\beta)}\,\mathrm{d}t,$$
$$= \frac{k}{T} + \sum_{i=1}^{e} \frac{-h_i}{D(2 - \beta)T}$$
$$\left[-D(1 - \beta)t \Big|_{t_{i-1}}^{t_i} + Q^{1-\beta} \right]^{(2-\beta)/(1-\beta)}.$$

Substituting the value of T from (5), and rearranging and simplifying terms gives:

$$\text{TIC} = \frac{kD(1 - \beta)}{Q^{1-\beta}} + \frac{h_1(1 - \beta)Q}{(2 - \beta)}$$
$$+ \sum_{i=1}^{e-1} \frac{(h_{i+1} - h_i)(1 - \beta)}{Q^{1-\beta}(2 - \beta)}$$
$$\times \left[Q^{1-\beta} - D(1 - \beta)t_i \right]^{(2-\beta)/(1-\beta)}. \tag{11}$$

To find the optimal order size Q^*, we set the derivative of TIC with respect to Q equal to zero. After simplification, we obtain:

$$-\frac{kD(1 - \beta)}{Q^{1-\beta}} + \frac{h_1 Q}{(2 - \beta)}$$
$$+ \sum_{i=1}^{e-1}(h_{i+1} - h_i)\left[Q^{1-\beta} - D(1 - \beta)t_i \right]^{1/(1-\beta)}$$
$$- \sum_{i=1}^{e-1} \frac{(h_{i+1} - h_i)(1 - \beta)}{Q^{1-\beta}(2 - \beta)}$$
$$\times \left[Q^{1-\beta} - D(1 - \beta)t_i \right]^{(2-\beta)/(1-\beta)} = 0. \tag{12}$$

If the entire inventory cycle happens to fall into the first period ($0 \leqslant T \leqslant t_1$), then $e = 1$, and the summations over i in (12) are empty. In that case, the optimum solution is simply obtained by substituting h_1 into (9) to calculate Q^*, and then substituting Q^* into (5) to calculate T. Obviously, a simple closed form solution for Q^* and T^* can be determined only if $T \leqslant t_1$. In general, the optimum solution must be determined by the following algorithm.

5.1. Solution algorithm

1. Substitute h_1 into (9) to determine $Q_{\max}$, and then substitute $Q_{\max}$ into (5) to determine $T_{\max}$. If $T_{\max} \leqslant t_1$, stop; the solution ($Q_{\max}, T_{\max}$) is optimal.
2. Substitute h_n into (9) to determine $Q_{\min}$, and then substitute $Q_{\min}$ into (5) to determine $T_{\min}$.
3. Depending on the values of $T_{\min}$ and $T_{\max}$, determine the possible periods that T may fall into (i.e., all feasible values of e).
4. For each feasible value of e, solve (12) numerically to determine the optimum value of Q. If Q corresponds to the correct period, it is considered realizable.
5. Using (11), calculate TIC for each Q_R and each $Q_i = Q(t_i)$.
6. Choose the value of Q that gives the lowest TIC.

5.2. Example 2

Use the same data given in Example 1 to determine Q^* and T^*, assuming that the increase in the holding cost is incremental.

Step 1: From previous example:

$Q_{\max} = 268$ units,

$T_{\max} = 0.426$ year,

Since $T_{\max} > t_1$, we must continue.

Step 2: Using (9)

$$Q_{\min} = \left[\frac{300(400)(1-0.1)(2-0.1)}{7}\right]^{1/1.9} = 224 \text{ units}.$$

Using (5)

$$T_{\min} = \frac{224^{0.9}}{400(1-0.1)} = 0.362 \text{ year}.$$

Step 3: Since $T_{\min}$ is in period 2 and $T_{\max}$ is in period 3, we need to develop total cost expressions TIC only for two possible end periods, $e = 2$ and 3.

Step 4: (a). Assuming $e = 2$

First, the cycle is assumed to end in the second period ($e = 2$). Thus, the cycle length T is assumed to be between $T_{\min}$ and t_2, i.e. the range of T in years is ($0.362 \leqslant T \leqslant 0.4$). Using (6), the corresponding Q range in units is ($224 \leqslant Q \leqslant 250$). Substituting the given values in (12), we obtain the following equation:

$$-\frac{108000}{Q^{0.9}} + \frac{5Q}{1.9} + \left[Q^{0.9} - 72\right]^{1/0.9}$$
$$-\frac{0.9}{1.9Q^{0.9}}\left[Q^{0.9} - 72\right]^{1.9/0.9} = 0.$$

Solving the above equation numerically by the secant method, using range limits $Q = 224$ and 250 as initial values, we obtain:

$Q = 212$ (realizable, since e
$= 2$ means $116 < Q < 250$, thus $Q_R = 212$).

(b) Assuming $e = 3$: Now, the cycle is assumed to end in the third period ($e = 3$). Thus, the cycle length T is assumed to be between t_2 and $T_{\max}$ ($0.4 < T \leqslant 0.426$). The corresponding Q range is ($250 < Q \leqslant 268$). Eq. (12) becomes:

$$-\frac{108000}{Q^{0.9}} + \frac{5Q}{1.9} + \left[Q^{0.9} - 72\right]^{1/0.9}$$
$$-\frac{0.9}{1.9Q^{0.9}}\left[Q^{0.9} - 72\right]^{1.9/0.9} + \left[Q^{0.9} - 144\right]^{1/0.9}$$
$$-\frac{0.9}{1.9Q^{0.9}}\left[Q^{0.9} - 144\right]^{1.9/0.9} = 0.$$

Solving by the secant method, with initial values $Q = 251$ and 268, we obtain:

$Q = 207$ (not realizable).

Step 5: The TIC should now be calculated for the two values of Q corresponding to the break points ($Q_1 = 116$, $Q_2 = 250$). Since Q_1 corresponds to $e = 1$, TIC(116) is obtained by (8) as:

TIC(116) = 1772.39.

Now, we use (11) to calculate TIC for $Q_R = 212$ and $Q_2 = 250$ (both corresponding to $e = 2$):

$$\text{TIC}(Q) = \frac{108000}{Q^{0.9}} + \frac{4.5Q}{1.9}$$
$$+ \frac{0.9}{1.9Q^{0.9}}\left[-72 + Q^{0.9}\right]^{1.9/0.9},$$

TIC(212) = 1388.58,
TIC(250) = 1369.86.

Step 6: The optimum solution is given by

$Q^* = 250$,

$T^* = 0.4$ year,

$\text{TIC}^* = \$1369.96/\text{year}$.

The above minimum TIC is lower than the value of 1460.43 obtained in Example 1. This is expected as incremental increase in holding cost is less costly than retroactive increase.

6. Conclusions and suggestions

A model has been presented of an inventory system with stock-dependent demand, in which the holding cost is a step function of storage time. Two types of holding cost variation in terms of storage time have been considered: retroactive increase, and incremental increase. Simple optimization algorithms have been developed, and numerical examples have been solved. Based on the formulas developed, it can be concluded that both the optimal order quantity and the cycle time decrease when the holding cost increases. As the shape parameter β increases, however, the optimal order quantity increases while the cycle time decreases. Moreover, the optimal order quantity and the EOQ are equal when $\beta = 0$.

The model presented in this study provides a basis for several possible extensions. For future research, this model can be extended to accommodate planned shortages, variable ordering costs, and

non-instantaneous receipt of orders. Another extension possibility would be to consider the holding cost as a *decreasing* step function of storage time. The case of the increasing holding cost considered in this paper applies to company-owned storage facilities, and particularly to perishable items that require extra care if stored for longer periods. A decreasing holding cost step function is applicable to rented storage facilities, where lower rent rates are normally obtained for longer-term leases.

Acknowledgment

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