

Modeling health care facility location for moving population groups

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Abstract

Locating public services for nomadic population groups is a difficult challenge as the locations of the targeted populations seasonally change. In this paper, the population groups are assumed to occupy different locations according to the time of the year, i.e., winter and summer. A binary integer programming model is formulated to determine the optimal number and locations of primary health units for satisfying a seasonally varying demand. This model is successfully applied to the actual locations of 17 seasonally varying nomadic groups in the Middle East. Computational tests are performed on different versions of the model in order to analyze the tradeoffs among different performance measures.

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1. Introduction

Facility location studies are generally devoted to the location of a set of resource or service facilities to optimally serve a given set of existing customer or demand facilities. Hale [1] provides an overview and a historical perspective of location science. It has been mostly assumed that the existing facilities or demand nodes (consumers, population sites, etc.) have unique and predefined locations. Thus, most of the work carried out so far assumes static locations where the demand over time is fixed as well as the physical locations of the facilities to be opened. In the area of health care facility location, this static approach has recently been exemplified by Rahman and Smith [2] and Verter and Lapierre [3].

In the field of dynamic location, traditional assumptions of fixed demands and locations are challenged by considering continuously or periodically changing demand volumes or locations. Wesolowsky [4] considers a single-facility location problem where demands vary in a fixed number of epochs. Wesolowsky and Truscott [5] introduce a dynamic multi-facility, multi-period location–allocation model to solve a problem where demand volumes change over time. Drezner and Wesolowsky [6] consider the problem where the demands are allowed to change a finite number of times. Abdel-Malek [7] determines the dynamic location of a moving service facility among a set of moving existing demand facilities, in order to minimize the total squared travel distance. Puerto and Rodriguez-Chia [8] develop an extension of the Weiszfeld algorithm to solve the dynamic Weber problem, in which the fixed demand nodes are replaced by trajectories (functions of time).

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In several countries it is not unusual for nomadic populations to periodically move and change their residence locations according to weather conditions or cultural and local traditions. As populations are not present in all sites during all seasons, decisions on whether or not to operate certain facilities during a given season should be taken. This situation describes the difficulty that local governments face when they try to provide public services to nomadic populations. Clearly these populations are entitled to all kinds of public services, such as primary health care, veterinary services and schooling for children. Providing good primary health care services for nomadic populations is a key step towards constructing a global welfare system integrating all levels of healthcare support. However, the key question is where to locate such facilities to best serve seasonally moving nomadic groups.

The remainder of this paper is organized as follows. Bedouin seasonal movement is described in Section 2. The problem's assumptions and formulation are respectively provided in Sections 3 and 4. A case application and computational results are presented in Section 5. Finally some conclusions are presented in Section 6.

2. Bedouin seasonal movement

In this work we discuss the possibility of opening fixed public facilities while the populations to be served occupy different location sites in specific periods (seasons) of the year, for instance winter or summer seasons. We assume that the predetermined number of facilities to be operated is known prior to finding the locations. Thus, we focus on finding the optimal location for primary health units that will minimize the total cost of serving all population groups during the different seasonal periods. Looking first at the characteristics of movements throughout the year should help in designing efficient healthcare systems for nomadic populations.

Bedouin is a term that refers to the nomadic desert dwellers in Saudi Arabia and neighboring countries. Although many of these Bedouin have now settled down, a large number still pursues a nomadic or semi-nomadic lifestyle. The Bedouin are pastoralists, whose survival depends on the animals they keep such as camels, goats and sheep. In order to feed their livestock, they have to move to different locations in the desert in search of water and fresh pasture. However, contrary to popular belief, their movement is not continuous or random, but rather follows a seasonal pattern to areas of higher rainfall [9]. During summer, they have to be close to permanent water sources because the animals need to drink more often.

During the annual migration cycle, the Bedouin usually move between two main locations for the summer and winter seasons. Cordes and Scholz [10] found that the Bedouin of the United Arab Emirates (UAE) and the Sultanate of Oman used seasonal or periodic movements between summer and winter pasture areas (SPA and WPA) to exploit the slight variations in climatic and physical characteristics of the territory. Usually SPA are located near oases to guarantee water and pasture supplies and suitable climatic conditions. On the other hand, WPA are located in *wadis* (depressions surrounded by sand dunes) where there are water holes, wells, or dams. The SPA are occupied during the date harvest period (May to October), while the WPA are occupied for the rest of the year.

3. General assumptions for the problem

In our study we assume that the physical locations of the population groups to be served are known for all periods (seasons) and are distributed on the plane. We consider opening a given number of fixed facilities (health care units) but only a predetermined subset of them will be operated during each season k . Trade-offs should be made between the total number of open facilities p and the number to operate during each period p_k depending on the opening costs and the seasonal running costs. Some more specific assumptions are made to describe the approach that we consider in this work.

1. Existing demands for services A_k (set of population sites) are located according to the given season k of the year.
2. There is a minimum service threshold L_j , i.e., total population to be served per season, below which a health unit will not be operated.
3. This is a discrete location problem, which can be defined on a graph. Population areas are aggregated to form a subset of the nodes of the network.
4. Potential sites for the location of the health care units are pre-specified and are all located at the vertices of the network. Euclidean distance measures are used between all locations.
5. The opening costs for health units depend on the location.
6. The operating costs for health units depend among other factors on the location and the season of the year.

In this problem the populations are assumed to occupy different locations in different seasons of the year. These seasonal residences are known and stably located and used over the years. However, small parts of each population group may choose an alternative summer or winter location different from the rest of the group. Therefore, the population size of each group may slightly vary with the seasons. Moreover, the operating cost at an open site may vary with the seasons due to changes in the weather conditions (e.g., air conditioning costs).

4. Problem formulation

The model introduced below represents the problem of opening a maximum number of p facilities from which we choose to serve the different population locations depending on seasonal demand. From the total set of opened sites, only a subset of facilities with a specified number, $p_k \leq p$, will be operated during a given seasonal period k . The necessary parameters to formulate the model are listed below.

Input parameters:

i, I	set of existing demand (population) groups, all assumed to be located at specific vertices of the graph
j, J	set of potential locations for the health care units
k, K	number of seasonal periods per year
p_k	number of units to be actively operated during a given season k
p	maximum number of health care units (from J) to open
w_{ik}	population size at demand node i during season k
t_{ijk}	transportation cost resulting from allocating demand node i to health care unit at j during season k
L_j	minimum workload required to open a health care unit at location j
f_j	cost of opening a health care unit at location j
o_{jk}	cost of operating a health care unit at location j during season k

Decisions variables:

x_{ijk}	1, if demand node i is assigned to unit at node j during season k ; 0 otherwise
y_j	1, if a facility is opened at location j ; 0 otherwise
u_{jk}	1, if an open facility at j is operated during season k ; 0 otherwise

Integer programming model:

$$\text{Min } Z = \sum_{i \in I} \sum_{j \in J} \sum_{k=1}^K w_{ik} t_{ijk} x_{ijk} + \sum_{j \in J} f_j y_j + \sum_{k=1}^K \sum_{j \in J} o_{jk} u_{jk} \quad (1)$$

Subject to

$$\sum_{j \in J} x_{ijk} = 1, \quad \forall i \in I, \quad \forall k \in K, \quad (2)$$

$$x_{ijk} \leq u_{jk}, \quad \forall i \in I, \quad \forall j \in J, \quad \forall k \in K, \quad (3)$$

$$u_{jk} \leq y_j, \quad \forall j \in J, \quad \forall k \in K, \quad (4)$$

$$\sum_{j \in J} y_j \leq p, \quad (5)$$

$$\sum_{j \in J} u_{jk} \leq p_k, \quad \forall k \in K, \quad (6)$$

$$\sum_{i \in I} w_{ik} x_{ijk} \geq L_j u_{jk}, \quad \forall j \in J, \quad \forall k \in K, \quad (7)$$

$$x_{ijk}, y_j, u_{jk} \in \{0, 1\}. \quad (8)$$

The objective (1) is to minimize the total cost, which includes three components: (i) the traveling cost of all the members of the population groups in all seasons, (ii) the fixed cost of opening all health care units, and (iii) the costs of operating a given number of the open facilities during each season. Of course, the opening costs may sometimes not be taken into consideration. The first constraint (2) shows that each population center must be assigned to a facility at each season (summer or winter for instance). In constraint (3), population groups are only assigned to open facilities, and constraint (4) shows that a facility can only operate if it is opened. The two constraints (5) and (6) specify the total number of health care units to open and to operate during each period. Constraint (7) describes the workload threshold that is required to consider operating any facility during season k .

The model as introduced allows decision makers to pre-specify a number of operated facilities during the different seasons while finding their best locations. If $k=1$ and $p_k = p$ for all period k then the description of the problem can be reduced to the *uncapacitated facility location* (UFL) problem. Since Cornuejols et al. [11] have shown that UFL is an NP-hard problem, we deduce that our problem as described is also NP-hard. The present formulation may also be viewed as a multi-period version of the UFL. As described, it is a static formulation tackling a dynamic problem because of the presence of the different seasonal demands. Since the location of the demand over time is known in advance, we do not deal with the dynamic facility location problem considered by Van Roy and Erlenkotter [12], in which the location in the next period depends on the previous location.

5. Actual application and computational results

The above model was applied to the populations of 17 nomadic groups in the UAE and Oman as shown in Fig. 1, adapted from Cordes and Scholz [10]. Each group alternates between a summer and a winter location as specified in the map. Therefore, our application of the model involves two seasons, summer and winter ($K=2$), where $k=1$ designates summer, and $k=2$ designates winter. We had to estimate the population sizes of the groups as current census data are not available. Table 1 shows the different data used for our application's model, including the population sizes and their location coordinates.

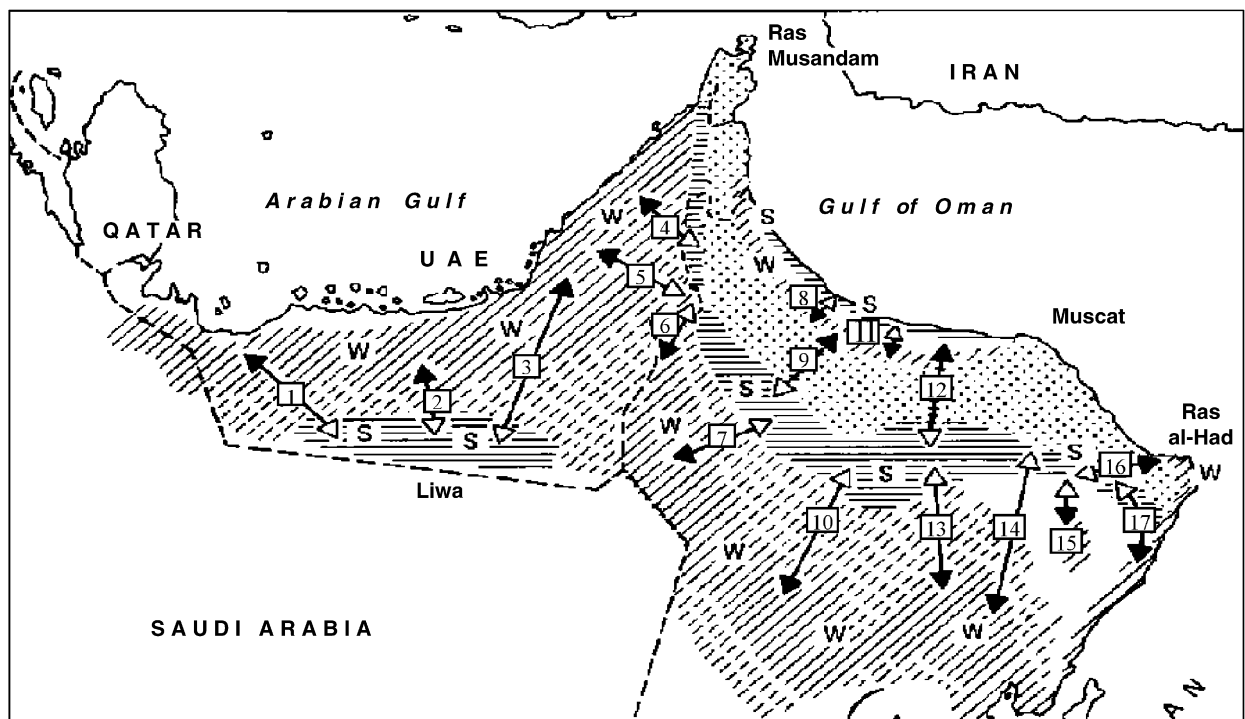


Fig. 1. Summer and winter locations of 17 Bedouin groups in the United Arab Emirates and the Sultanate of Oman.

Table 1

Location coordinates and population sizes in summer and winter

Population group <i>i</i>	Summer location coordinates		Winter location coordinates		Summer population	Winter population
	<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>	Size w_{i1}	Size w_{i2}
1	236	360	169	289	525	401
2	306	356	296	298	337	641
3	352	364	400	225	594	526
4	496	197	455	156	542	345
5	485	238	423	202	530	364
6	491	247	470	293	680	431
7	548	344	478	375	479	444
8	595	239	576	261	359	429
9	550	323	595	271	674	383
10	604	386	557	491	696	400
11	638	265	633	292	396	394
12	633	366	673	281	382	583
13	667	384	672	486	339	394
14	736	372	709	508	491	447
15	761	395	761	433	428	606
16	768	391	829	378	673	624
17	797	396	812	465	305	439

Table 2

Coordinates and operating conditions of the potential facility locations

Facility site <i>j</i>	Coordinates		Workload threshold L_j	Opening cost f_j	Operating costs	
	<i>x</i>	<i>y</i>			Summer o_{j1}	Winter o_{j2}
1	169	289	3023	115700	110000	66000
2	306	356	3518	125000	56000	120000
3	400	225	1633	270000	130000	62000
4	496	197	3297	110000	65000	130000
5	557	491	3204	175000	100000	60500
6	550	323	2883	230000	46000	110000
7	633	292	1761	110000	120000	56000
8	667	384	4103	170000	46000	110000
9	829	378	1887	180000	120000	62000
10	768	391	2472	240000	62000	140000

Ten potential sites for locating the health care units were chosen from among the 34 existing summer and winter locations of the nomadic groups. These potential sites, whose data are shown in Table 2, represent a mixture of summer locations (even values of j) and winter locations (odd values of j). The traveling costs c_{ijk} were computed as the straight-line distances between the existing locations of each population i and the potential site j for each season k . The operating costs are estimated and assumed to be different for the two seasons. If for instance a winter site is operated during a summer period, its operating costs will be greater as the populations have already moved away from the area.

To test the model, we did not focus on solving large instances of the problem. We rather tried to describe the properties of the model, looking at the behavior of the optimal solution under different conditions. A C⁺⁺ program was used to generate the model in the format required by LINGO 6.0, which was used to solve different instances of the model. Using a Pentium 4 PC with 2.00 GHz CPU, the run times ranged between 1 and 2.5 s. The program was run for different values of the vector (p, p_1, p_2) in order to analyze the optimal solutions and their general behavior depending on the considered parameters. The problem was solved both with and without considering the opening costs given in Table 2. Without opening costs, the model consists of 350 variables and 1750 constraints. Including opening costs, the number of variables becomes 370 with 1810 constraints. We also considered the possibility for the decision makers to state

Table 3
Zero opening cost and limit on the maximum number of operated facilities

(p, p_1, p_2)		(3, 3, 2)	(4, 3, 2)	(5, 3, 2)
Sites selected		3, 6, 10	3, 6, 8, 10	3, 6, 8, 10
Sites	$k = 1$	3, 6, 10	3, 6, 10	3, 6, 10
operated	$k = 2$	3, 10	3, 8	3, 8
Total cost		2,068,851	2,040,853	2,040,853
(p, p_1, p_2)		(4, 3, 3)	(5, 2, 3)	(6, 4, 3)
Sites selected		3, 6, 7, 10	3, 6, 7, 9, 10	3, 6, 7, 9, 10
Sites	$k = 1$	3, 6, 10	6, 10	3, 6, 10
operated	$k = 2$	3, 7, 10	3, 7, 9	3, 7, 9
Total cost		1,880,812	1,851,694	1,834,587

Table 4
Zero opening cost results

(p, p_1, p_2)		(5, 3, 2)	(5, 2, 3)	(6, 3, 3)	(7, 4, 3)	(7, 3, 4)
(a) With limits on maximum number of operated facilities						
Sites selected		3, 6, 8, 10	3, 6, 7, 9, 10	3, 6, 7, 9, 10	3, 6, 7, 9, 10	3, 6, 7, 9, 10
Sites	$k = 1$	3, 6, 10	6, 10	3, 6, 10	3, 6, 10	3, 6, 10
operated	$k = 2$	3, 8	3, 7, 9	3, 7, 9	3, 7, 9	3, 7, 9
Total cost		2,040,853	1,851,694	1,834,587	1,834,587	1,834,587
(p, p_1, p_2)		(5, 3, 2)	(5, 3, 3)	(6, 3, 3)	(7, 4, 3)	(7, 3, 4)
(b) With limits on minimum number of operated facilities						
Sites selected		3, 6, 7, 9, 10	3, 6, 7, 9, 10	3, 6, 7, 9, 10	3, 6, 7, 9	3, 6, 7, 9, 10
Sites	$k = 1$	3, 6, 10	3, 6, 10	3, 6, 10	3, 6, 7, 9	3, 6, 10
operated	$k = 2$	3, 7, 9	3, 7, 9	3, 7, 9	3, 7, 9	3, 7, 9, 10
Total cost		1,834,587	1,834,587	1,834,587	2,101,842	2,301,061

Table 5
Non-zero opening costs and maximum limit on the number of operated facilities

(p, p_1, p_2)		(3, 3, 2)	(3, 2, 3)	(5, 3, 2)
Sites selected		4, 8	4, 8	4, 8
Sites	$k = 1$	4, 8	4, 8	4, 8
operated	$k = 2$	4, 8	4, 8	4, 8
Total cost		2,634,734	2,634,734	2,634,734
(p, p_1, p_2)		(5, 2, 3)	(6, 4, 3)	(6, 4, 4)
Sites selected		4, 8	3, 7, 9,	3, 7, 9,
Sites	$k = 1$	4, 8	3, 7, 9,	3, 7, 9
operated	$k = 2$	4, 8	3, 7, 9	3, 7, 9
Total cost		2,634,734	2,624,203	2,624,203

either the maximum number ($\sum_{j \in J} u_{jk} \leq p_k, \forall k \in K$) or the minimum number ($\sum_{j \in J} u_{jk} \geq p_k, \forall k \in K$) of facilities to be operated. The different computational results are summarized as in Tables 3–5.

Table 3 shows the computational results for the case without opening cost ($f_j = 0$ for all $j \in J$) and a maximum limit on the number of operated facilities ($\sum_{j \in J} u_{jk} \leq p_k, \forall k \in K$). From Table 3 it can be seen that increasing the number of open facilities p will improve the solution if we keep the same number of operated units during the two seasons. This result was expected as the pair of constraints (3) and (4) forces the model to open at most $\min\{p, p_1 + p_2\}$ facilities.

Table 3 also shows that the solution to this problem is not equivalent to the solution of two separate single period UFL problems. Indeed the UFL solutions would be consistently the best sets of p_k sites irrespective of the value of p . This is not the case for our model, as solutions (3, 3, 2) and (4, 3, 2) are showing different optimal solutions for the two seasons.

Looking at the way population groups are allocated to the open units, we find that they are not necessarily allocated to the same health care unit during both winter and summer seasons. That is the case, for example, for demand node 14, when $(p, p_1, p_2) = (5, 3, 2)$; it is allocated to unit 6 for season 1 and to unit 8 for season 2. One can notice that it is not unusual at the optimum solution to operate a unit located out of season. That is, to operate in summer a unit that is a winter population site. The potential site 3 is always open and operates during both winter and summer except for the case $(p, p_1, p_2) = (5, 2, 3)$.

We have also tested the possibility for the decision makers to state the number of facilities to be operated either as a minimum or a maximum requirement. Table 4 provides some comparisons on that respect. The two options are not totally equivalent. Expressing a minimum requirement constraint ($\sum_{j \in J} u_{jk} \geq p_k, \forall k \in K$) tends to open more facilities, which minimizes the total traveled distances. However, this approach does not always guarantee a best solution.

Table 5 shows the results of the model solution with facility opening costs and a maximum limit on the number of operated facilities. Adding opening costs to the model tends to increase the total cost although it reduces the number of open facilities. This can be seen by comparing both the total cost and the number of open facilities for similar cases in Tables 3 and 5.

6. Conclusions

A facility location model has been presented for seasonally moving populations, such as for nomadic or Bedouin populations. The model allows decision makers to consider the trade off between opening costs and operating cost of public service facilities when population are allowed to reside in different locations during different periods of the year. All opened facilities must satisfy a minimum workload requirement during the season in which they are operated. Computational tests have been performed on different versions of the problem in order to analyze the main characteristics of the model.

The model could be extended to allow budget planning constraints such as having a budget related limitation to determine the number of operated facilities. Another possible extension is to consider mixing fixed health care units with mobile, removable and relocatable units to match the populations' living style. A maximum distance threshold for the traveling distances could be considered to limit the costs of the population traveling to access the health care services. These issues will be addressed in future research.

Another extension route would be to integrate the model in a planning support system that will assist on key strategic decision planning processes. This model constitutes a first step towards the design of a complete healthcare solution for moving populations. Commonly integrated healthcare services combine primary health units to district and/or regional hospitals. The complete design of such systems is a challenge that will require the solution of multistage facility location problems. The model presented here is intended to be one of these stages.

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References

- [1] Hale TS. Location science research. *Annals of Operations Research* 2003;123:21–35.
- [2] Rahman S-U, Smith DK. Deployment of rural health facilities in a developing country. *Journal of the Operational Research Society* 1999;50: 892–902.
- [3] Verter V, Lapierre SD. Location of preventive health care facilities. *Annals of Operations Research* 2002;110:123–32.
- [4] Wesolowsky GO. Dynamic facility location. *Management Science* 1973;19:1241–8.
- [5] Wesolowsky GO, Truscott W. The multiperiod location–allocation problem with relocation of facilities. *Management Science* 1975;22:57–65.
- [6] Drezner Z, Wesolowsky GO. Facility location when demand is time dependent. *Naval Research Logistics* 1991;38:763–77.

- [7] Abdel-Malek LL. Optimum positioning of moving service facility. *Computers and Operations Research* 1985;12:437–44.
- [8] Puerto J, Rodriguez-Chia AM. Location of moving service facility. *Mathematics of Operations Research* 1999;49:373–93.
- [9] Al-Eisa A. Changes and factors affecting Bedouin movement for grazing, in *Livestock: Coping with Drought*, an electronic conference organized by the Food and Agriculture Organization of the United Nations, 1998, (<http://www.odi.org.uk/pdn/drought/>).
- [10] Cordes R, Scholz F. *Bedouin, Wealth, and Change: A Study of Rural Development in the United Arab Emirates and the Sultanate of Oman*. United Nations University Press, 1980. (<http://www.unu.edu/unupress/unupbooks/80143e/80143E00.htm>).
- [11] Cournéjols G, Nemhauser G, Wolsey L. The uncapacitated facility location problem. In: Mirchandani PB, Francis RL, editors. *Discrete Location Theory*. New York: Wiley; 1990. p. 119–72 (Chapter 3).
- [12] Van Roy TJ, Erlenkotter D. A dual-based procedure for dynamic facility location. *Management Science* 1982;28:1091–105.