Optimizing a Library's Loan Policy: An Integer Programming Approach

Hesham K. Al-Fares
Systems Engineering Department, King Fahd University of Petroleum and Minerals, P.O. Box 2003, Dhahran 31261, Saudi Arabia. E-mail: hesham@ccse.kfupm.edu.sa

Providing the best service in order to satisfy users is the main objective of any library. The loan policy is a major tool available to achieve this objective. Previous studies have all focused on the loan period, ignoring the loan policy's other equally important aspect, which is the maximum number of books a user can borrow. Moreover, only book availability has been used as a measure of user satisfaction. User satisfaction with either the length of the loan period or the number of books allowed to be borrowed has been largely overlooked. This article combines those relevant components into a more complete model. An easy-to-solve integer programming model is formulated, whose solution yields the optimum loan period and optimal limit on the number of books that can be borrowed to maximize user satisfaction. A case study of an actual university library is presented.

1. Introduction

With resources shrinking, and collections and the number of users ever growing, library management is an increasingly difficult task. Hindle and Buckland (1976) maintain that the main objective of a library, as a service facility, is to satisfy its users. They classify library management into the following decision areas: Acquisition, collection control, user education, and administrative systems. The major tool available to library management for achieving its user satisfaction objective is the collection control, or loan policy. While the loan policy has a large influence on user satisfaction, it is also directly under the control of the librarians.

According to Buckland (1975, p. 75), three regulations constitute a loan policy: (1) The maximum loan period, (2) the maximum number of books that can be borrowed by the same user, and (3) the number of renewals allowed. For the sake of completeness, we can add a fourth regulation: One that concerns reservations and recalls. Of these four regulations, the first two are much more important. Buckland (1972) finds that while there is a strong tendency for books to be kept out until their due date, the great majority of loans are not renewed. Somewhat surprisingly, even the length of the official loan period has little effect on the frequency of renewals. Moreover, Buckland (1975, p. 91) also observes that reservations are made only on a small fraction of instances when books are not immediately available.

This article is mainly concerned with the two more important aspects of the loan policy: (1) The length of loan period, and (2) the number of books allowed to be borrowed. The objective is to maximize user satisfaction with both of them, in addition to satisfaction with book availability. The article is organized as follows. First, a review of previous relevant literature is given. Then, the integer programming model of the problem is formulated. Subsequently, considerations for implementing the model are discussed. Next, a case study is presented, and availability percentages are compared with previously published values. Finally, conclusions and recommendations are given.

2. Literature Review

The application of operations research (OR) techniques to library problems was pioneered by Morse (1968, 1979). In his book, *Library Effectiveness: A Systems Approach*, he relies primarily on queuing theory and Markovian processes to model circulation and book use. Morse also uses queuing theory to analyze loan policies, in particular the effect of reducing the loan period on satisfying circulation demand. For library effectiveness, the criterion that Morse uses is the ability to provide service (that is, the availability of desired books) to users.

Using computer simulation, significant contributions have been made by the library research group at the University of Lancaster (Buckland, 1972, 1975; Hindle & Buckland, 1976). Their main objective is maximizing book availability, which they consider to be the result of three

Received September 25, 1996; revised August 1, 1997; accepted October 30, 1997.

interacting factors: (1) The frequency of demand, (2) the length of the loan period, and (3) the amount of duplication. Two measures of effectiveness are defined: (1) Satisfaction level: The average immediate availability for a given time duration; and (2) collection bias: The average availability of the 10% most requested books. The group's main recommendation is to implement a variable loan policy, in which the most popular books are subject to a shorter loan period.

Several studies related to loan period, availability, and satisfaction have been conducted at Case Western Reserve University. Kantor (1976b) proposes using document exposure time and user satisfaction level as a combined measure of library performance. Kantor (1976a) uses a branching diagram to describe how availability is reduced by four factors: Lack of acquisition, circulation, misshelving, and user error. Shaw (1976a) develops a simulation model that evaluates the effect of the loan period on user satisfaction, which is measured by book availability and delay associated with recalls. Shaw (1976b) conducts a survey of loan period distribution in academic libraries. Saracevic, Shaw, and Kantor (1977) use a branching approach to analyze the causes of user dissatisfaction, including the loan period.

An approach employing basic queuing theory has been used by Goyal (1970) to determine the optimal loan periods for periodicals. Two models are proposed, both considering readers as customers, periodicals as servers, and the loan period as service time, which is considered exponentially distributed as a simplifying assumption. The objective of the first model is to minimize the total cost of providing the service plus the cost of waiting. The objective of the second model is to maximize customer satisfaction, assumed to be a function of the loan period, book availability, and waiting time.

Bookstein (1975) develops a more sophisticated analytical model based on queuing theory in order to determine optimal loan periods. The model assumes that no queues are allowed to form (no reservations permitted), but it considers the effect of user satisfaction with the loan period. Considering transactions at the circulation desk as generating costs, the objective of the model is to minimize the number of these transactions. Alternative objective functions, such as maximizing book availability or maximizing the probability of successful book use, are also discussed.

Several studies deal with the loan policy and its relationship to various aspects of the library system. Burkhalter and Race (1968) analyze the effect of renewals, overdues, and other factors on the optimal loan period. Bruce (1975) models the library loan dynamics as a four-state Markovian system, relating the loan period to this system. Yagello and Guthrie (1975) examine the increase in circulation resulting from reducing the loan period. Sinha and Clelland (1976) present a general model for collection control, which can be extended to include variations in the loan period. Burrell (1980) proposes, and Hindle and Worthington (1980) discuss, simple stochastic models for library loans which explain some empirical circulation frequency distributions. Burrell and Fenton (1994) develop a modified model of library circulation which accounts for book unavailability for borrowing while it is out on loan. Goehlert (1978) analyzes the influence on availability by several factors, including book reshelving, search, circulation, on order, not owned, duplication, age, and subject. Goehlert determines that circulation is the most important factor. Kantor (1979) presents a comprehensive review of library operations research up to 1979. Kraft and Boyce (1991) provide the most recent and extensive bibliography of library OR literature.

3. The Integer Programming Model

The literature review confirms the importance of loan policy for library user satisfaction. The four elements of the loan policy have been identified as: Loan period, number of books allowed, renewals, and reservations or recalls. The first two are shown to be more important; however, all of the reviewed research seems to focus only on the loan period. Although the maximum number of books to be borrowed is also significant for achieving user satisfaction, it has been apparently overlooked in the literature. The model to be presented here considers the two major elements of the loan policy: (1) The length of the loan period, and (2) the maximum number of books allowed to be borrowed.

User satisfaction appears to be a common objective based on all the literature surveyed. For measuring satisfaction, most articles simply choose immediate availability—the probability of finding a required book available on the shelf. Variations include availability of the most popular books, satisfaction with the loan period and the waiting time, and some cost-based objectives. We believe that immediate availability is not an adequate measure of user satisfaction, which must include satisfaction with both the loan period and the maximum number of books that are allowed to be borrowed. Just as a user becomes dissatisfied when books are not available, he or she will be dissatisfied if the loan period is too short or the books he or she is allowed to borrow are too few. Thus, three measures of satisfaction are used in this article:

$s1$ = satisfaction with the loan period,
$s2$ = satisfaction with the maximum number of books allowed to be borrowed, and
$s3$ = satisfaction with book availability.

3.1. Model Assumptions

1. User satisfaction, for all three types, is defined to be a ratio of satisfied demands to total demands. This is consistent with approaches used in the literature. For example, Hindle and Buckland (1976) define satisfaction level as the *proportion* of demands which can be immediately satisfied.

2. User demands, for the number of books or the length of loan period, will not be affected by changes in the loan policy. If this is not the case, then according to Hindle and

Buckland (1976), one would expect the renewal probability to change with the loan period. It was mentioned earlier that, to the contrary, the length of the official loan period is seen to have negligible influence on the frequency of renewals.

3. As discussed previously, availability is affected by many factors, such as the loan period, duplication, and demand, which have been identified by Buckland (1975), Kantor (1976a), and Goehlert (1978). However, both Buckland and Goehlert conclude that the loan policy has, by far, the most significant impact on availability. Therefore, all other factors are assumed constant, and thus their effect is ignored, as we concentrate on the relationship between the loan policy and availability.

4. For the model to be realistic, $s3$ should be considered to have a more significant impact on overall user satisfaction than either $s1$ or $s2$. The level of dissatisfaction resulting from book unavailability should be greater than that resulting from not getting all the borrowing time or number of books desired. If books are not available, the user will not be concerned about how many to borrow or how long to keep them. Determining the relative weight of the three satisfaction components requires further research well beyond the scope of this article. Therefore, it will be assumed here that $s3$ is much more important than either $s1$ or $s2$.

3.2. Model Parameters

Let

I = upper limit on the official loan period in weeks
J = upper limit on the maximum number of books a user is allowed to borrow
i = length of the official (maximum) loan period in weeks, $i = 1, \ldots, I$
j = maximum number of books allowed to be borrowed by the same user, $j = 1, \ldots, J$
w = number of loan weeks desired by the user, $w = 1, \ldots, I$
b = number of books desired to be borrowed by the user, $b = 1, \ldots, J$
a = number of books available, out of those desired by the user, $a = 0, \ldots, b$.

Obviously, $s1$ (satisfaction with loan period i) is a function of i, and $s2$ (satisfaction with number of books j) is a function of j. Since availability is affected by the loan period and the number of books allowed, $s3$ (satisfaction with book availability) is a function of both i and j. A user is partially satisfied if the loan policy does not allow him all the borrowing time or number of books he needs; in this case, we assume that satisfaction is the ratio of what is needed to what is allowed. On the other hand, a user is completely satisfied if the policy meets or exceeds his needs. Keeping in mind that satisfaction cannot exceed 100%, $s1$, $s2$, and $s3$ can be defined for each user as follows:

$$s1 = \min\left\{1, \frac{i}{w}\right\} \tag{1}$$

$$s2 = \min\left\{1, \frac{j}{b}\right\} \tag{2}$$

$$s3 = \frac{a}{b}. \tag{3}$$

Surveys of user needs must be conducted to determine user borrowing needs and current book availability. From these, probability distributions can be constructed for both the loan period and number of books required for borrowing. Given

f_w = the probability that a user needs w weeks
g_b = the probability that a users needs b books.

The average satisfaction with the loan period, $S1_i$, and with the number of books, $S2_j$, can be calculated for any given policy as follows:

$$S1_i = \sum_{w=1}^{I} f_w^* \min\left\{1, \frac{i}{w}\right\}, \; i = 1, \ldots, I \tag{4}$$

$$S2_j = \sum_{b=1}^{J} g_b^* \min\left\{1, \frac{j}{b}\right\}, j = 1, \ldots, J. \tag{5}$$

Another approach is used to calculate average availability $S3$. It is easier to calculate book unavailability ($S3' = 1 - S3$), since it is directly related to the loan period i and the maximum number of books j. A logical assumption is that unavailability—the probability of not finding a needed book—depends on the average borrowing time W_i times the average number of borrowed books B_j. If the needed time w is less than the official loan period i, the user will keep books for w weeks; if w is greater than i, he/she will borrow them for only i weeks. Similarly, if the number of needed books b is less than the maximum number allowed j, the user will borrow b books; if b is greater than j, he will borrow only j books. Noting that the loan period cannot exceed i, and that the number of books cannot exceed j, W_i and B_j can be estimated for each policy as shown below.

$$W_i = \sum_{w=1}^{I} f_w^* \min\{w, i\}, \; i = 1, \ldots, I \tag{6}$$

$$B_j = \sum_{b=1}^{J} g_b^* \min\{b,j\}, j = 1, \ldots, J \tag{7}$$

For each policy, average unavailability $S3'$ is assumed proportional to the product of the average borrowing time W times the average number of borrowed books B. Let us define α as the average demand per book per week. If one book is checked out for one week, there will be an average

of α unavailabilities (unsatisfied demands). In general, if on average B_j books are checked out for W_i weeks, the average unavailability is $\alpha W_i B_j$. Thus, for each library, there is a demand constant α such that:

$$S3'_{ij} = \alpha W_i B_j,\ i = 1, \ldots, I,\ j = 1, \ldots, J. \quad (8)$$

3.3. Decision Variables

$$X_i = \begin{cases} 1 & \text{if the loan period is } i \text{ weeks} \\ 0 & \text{otherwise} \end{cases}, \quad i = 1, \ldots, I \quad (9)$$

$$Y_j = \begin{cases} 1 & \text{if the number of books is } j \text{ books} \\ 0 & \text{otherwise} \end{cases}, \quad j = 1, \ldots, J \quad (10)$$

3.4. Objective Functions

The objective is to maximize average total satisfaction, which is the aggregate of $S1$, $S2$, and $S3$. First, $S3$ is multiplied by the constant C ($C > 1$) to indicate that it is much more important than $S1$ or $S3$. However, since $S3 = 1 - S3'$, maximizing the availability $S3$ is replaced by maximizing the negative of unavailability $S3'$.

$$\text{Maximize} \sum_{i=1}^{I} S1_i X_i + \sum_{j=1}^{J} S2_j Y_j - \text{C} \sum_{i=1}^{I} \sum_{j=1}^{J} S3'_{ij} X_i Y_j \quad (11)$$

or, from Equation 8:

$$\text{Maximize} \sum_{i=1}^{I} S1_i X_i + \sum_{j=1}^{J} S2_j Y_j - \text{C}\alpha \sum_{i=1}^{I} \sum_{j=1}^{J} W_i B_j X_i Y_j. \quad (12)$$

Here, α is a demand constant, defined by Equation 8, which must be calculated from the data for each library. First, calculate average unavailability $S3'$ for the current loan policy of n weeks and m books. For a sample of K users, $S3'$ and α are calculated as:

$$S3'_{nm} = 1 - (\sum_{k=1}^{K} \alpha_k / \sum_{k=1}^{K} b_k) \quad (13)$$

$$\alpha = \frac{S3'_{nm}}{W_n B_m}. \quad (14)$$

It can also be reasonably assumed, as indicated by Buckland (1972) and Shaw (1976b), that user needs for different loan periods have little effect on book return time, i.e., that users almost always wait until the books are due back. In this case, the average loan period W_i is equal to the official loan period i, and the objective function becomes:

$$\text{Maximize} \sum_{i=1}^{I} S1_i X_i + \sum_{j=1}^{J} S2_j Y_j - \text{C}\alpha \sum_{i=1}^{I} \sum_{j=1}^{J} i B_j X_i Y_j. \quad (15)$$

From the data for the *current* loan policy of n weeks and m books, the constant α is now computed by:

$$\alpha = \frac{S3'_{nm}}{nB_m}. \quad (16)$$

Both objective functions 12 and 15 are nonlinear, since they involve the product of X_i and Y_j. In order to solve by linear programming, the linearization technique developed by Watters (1967) for 0–1 variables must be applied first. Fortunately, this is not necessary, since the solution can be easily obtained by simple search.

3.5. Constraints

Since only one official loan period must be chosen, only one of the 0–1 X_i variables must be equal to 1, while the others must be equal to 0.

$$\sum_{i=1}^{I} X_i = 1 \quad (17)$$

A similar constraint must be included to make only one choice for the maximum number of books.

$$\sum_{j=1}^{J} Y_j = 1 \quad (18)$$

4. Implementation Considerations

Unfortunately, computerized circulation data do not usually show user needs with respect to the loan policy, nor contain information on book availability. An exit poll must be conducted at the circulation checkout counter to obtain these data from users after they have been through the library; only then can they supply information regarding book availability. A convenient approach for doing this is by means of a short questionnaire that the user is requested to fill out at the time of checking out books. It is important to emphasize the fact to users that we want to know their

TABLE 1. Loan period demands and calculations.

Maximum loan period in weeks i	1	2	3	4	5	6	7	8	9	10
% Demand for i weeks = $100 * f_i$	5	6	8	2	36	3	6	5	20	9
% Satisfaction with i weeks = $100 * S_i$	23.5	41.9	57.4	70.2	82.5	87.6	92.2	96.0	99.1	100
Average borrowing time in weeks W_i	1	1.95	2.84	3.65	4.44	4.87	5.27	5.61	5.90	5.99

actual needs, regardless of the official loan policy currently in force.

The input parameters to the model must be obtained by surveying users of the library. For each user, data must be collected on the needed number of weeks w, needed number of books b, and number of needed books available a. In determining the frequency of user demands for given numbers of books g_b, the number of needed books b must include books already on loan by the same user, reflecting total user needs. However, in calculating availability using Equations 3 or 13, books already on loan must be included, since we are interested only in immediate availability. The upper limits on the number of weeks I and number of books J must be set by the librarian, according to both user needs and practical considerations. Any given user needs that exceed any of these limits must be considered equal to the violated limit(s).

For obtaining the optimum solution, it is not necessary to use specialized integer programming software. Because of the simple 0–1 structure, it is more convenient to use spreadsheet packages such as Excel or Lotus. With simple spreadsheet techniques, one can develop an $I \times J$ matrix, in which each (i, j) cell contains the following value for the objective function:

$$S1_i + S2_j - C\alpha W_i B_j. \tag{19}$$

The cell with the highest value is chosen. Let this be cell (d, e), then the optimum loan policy calls for a maximum loan period of d weeks and a maximum of e books per user. Of course, W_i must be set equal to i for the second objective function 15. Although the integer programming model is not necessary for the solution, it provides a formal mathematical representation of the optimization problem. Moreover, it serves as a basis for more elaborate models with additional realistic considerations. For example, different loan policies for faculty, graduate students, and undergraduates can be optimized in an enlarged model.

5. Case Study

The proposed model was applied successfully at the central library of King Fahd University of Petroleum and Minerals in Saudi Arabia. There are three classes of users with three different loan policies: Faculty, graduate students, and undergraduate students. The study concentrated on the class of undergraduate students, as it was the largest group. The loan policy for undergraduates allows a maximum of seven books and 4 weeks. First, a questionnaire form was developed to obtain data on student needs and book availability. Students were asked to ignore the current loan regulations and express their actual needs by answering the following questions:

1. How many books do you *need* to borrow?
2. How many other books have you already borrowed and not yet returned?
3. Of the number needed (stated in Question 1), how many books were checked out by others?
4. How many weeks do you *need* to keep the borrowed books?

On the basis of students input and practical considerations, the maximum number to be considered of weeks I and books J were both set to 10. This agrees with Hindle and Buckland's (1976) survey which showed that loan periods in most university libraries do not exceed 10 weeks.

Table 1 shows the demand probabilities, percentage satisfaction, and average borrowing time for each official (maximum) loan period. Frequency of student needs for certain lengths of the borrowing time in weeks, taken from the survey, was used to obtain the demand probabilities. For each official loan period i, average percentage satisfaction with the number of weeks $S1_i$ was calculated using Equation 4. The average borrowing time in weeks W_i was computed by Equation 6.

Table 2 shows the demand probabilities, percentage satisfaction, and average number of borrowed books, for each limit on the number of books. Frequency of student needs for certain numbers of books was used to obtain the demand probabilities. For each limit on the number of books j, average percentage satisfaction with the number books $S2_j$ was computed using Equation 5. The average number of borrowed books B_j was calculated for each book limit j by Equation 7.

5.1. Objective Function 12

In order to apply the model, constants must be calculated based on the present loan policy and current book availability. First, using Equation 13 on the survey data, current average unavailability $S3'$ was determined to be 22.5%,

TABLE 2. Number of books' demands and calculations.

Maximum no. of books allowed j	1	2	3	4	5	6	7	8	9	10
% Demand for j books = $100 * g_j$	6	6	15	17	25	7	13	3	2	6
% Satisfaction with j books = $100 * S2_j$	27.5	48.9	67.4	80.9	90.1	94.3	97.4	98.6	99.4	100
Average no. of borrowed books B_j	1	1.94	2.82	3.55	4.11	4.42	4.66	4.77	4.85	4.91

which meant average availability is 77.5%. Tables 1 and 2 respectively show the average loan time and average number of borrowed books for the existing policy of 4 weeks and seven books ($W_4 = 3.65$, $B_7 = 4.66$). Equation 13 can now be used to calculate the demand constant for the first objective function 12.

$$\alpha = 0.225/(3.65) * (4.66) = 0.01323$$

Having calculated α, we can determine average book availability $S3_{ij}$, the probability of finding a needed book, for each limit on the loan period i and number of books j. First, average unavailability $S3'_{ij}$ is found by Equation 8, then $S3$ is set equal to $1 - S3'_{ij}$. The values obtained are displayed in Table 3.

Now, all the input parameters needed for the model have been calculated. Arbitrarily setting $C = 2$, the resulting binary integer programming model was then solved using the LINDO® linear programming package. The following optimal solution was obtained:

Maximum loan period i = 7 weeks
Maximum number of books j = 5 books
Percent average satisfaction = 100(S1+S2+S3)/3
= (92.2+90.1+72.3)/3
= 81.3%.

5.2. Objective Function 15

For the second objective function 15, the constant α is calculated by Equation 16, then average availability values $S3_{ij}$ are calculated as before and shown in Table 4.

$$\alpha = 0.225/4 * (4.66) = 0.01207$$

The optimal solution for the second objective, also obtained by LINDO®, is given below.

Maximum loan period i = 7 weeks
Maximum number of books j = 5 books
Percent average satisfaction = (92.2 + 90.1 + 71.9)/3
= 80.9%

5.3. Sensitivity Analysis

With both objective functions, the above solutions have been obtained by using the somewhat arbitrary value of (2.0) for C, the coefficient of $S3$, indicating that $S3$ is twice as significant as $S1$ or $S2$. For objective function 12, the above solution remains valid for values of this coefficient ranging from 2.0 to 2.2. For objective function 15, the solution is valid for coefficient values ranging from 1.8 to 2.1. This indicates that the solution is rather sensitive to the value of the coefficient. There is clearly a need to establish more reliable values for the coefficients of $S1$, $S2$, and $S3$. Therefore, a full-scale investigation of user opinions must be carried out to determine the relative influence of the three factors on overall satisfaction.

6. Comparison with Buckland's Values

It is interesting to note the remarkable similarity between the availability figures found by the new model, shown in Tables 3 and 4, and those determined by Buckland (1975).

TABLE 3. Percentage book availability $S3_{ij}$ for each policy with objective 12, assuming average borrowing time is i weeks.

	Books j									
Weeks i	1	2	3	4	5	6	7	8	9	10
1	98.7	97.4	96.3	95.3	94.6	94.2	93.8	93.7	93.6	93.5
2	97.4	95	92.7	90.8	89.4	88.6	88	87.7	87.5	87.3
3	96.2	92.7	89.4	86.7	84.6	83.4	82.5	82.1	81.8	81.6
4	95.2	90.6	86.4	82.9	80.2	78.7	77.5	77	76.6	76.3
5	94.1	88.6	83.4	79.2	75.9	74	72.3	72	71.5	71.2
6	93.6	87.5	81.8	77.1	73.5	71.5	70	69.3	68.8	68.4
7	93	86.5	80.3	75.3	71.4	69.2	67.5	66.8	66.2	65.8
8	92.6	85.6	79.1	73.7	69.5	67.2	65.4	64.6	64	63.6
9	92.2	84.9	78	72.3	67.9	65.5	63.6	62.8	62.2	61.7
10	92.1	84.6	77.7	71.9	67.4	65	63.1	62.2	61.6	61.1

TABLE 4. Percentage book availability $S3_{ij}$ for each policy with objective 15, assuming average borrowing time is i weeks.

	Books j									
Weeks i	1	2	3	4	5	6	7	8	9	10
1	98.8	97.7	96.6	95.7	95	94.7	94.4	94.2	94.1	94.1
2	97.6	95.3	93.2	91.4	90.1	89.3	88.8	88.5	88.3	88.1
3	96.4	93	89.8	87.1	85.1	84	83.1	82.7	82.4	82.2
4	95.2	90.6	86.4	82.9	80.2	78.7	77.5	77	76.6	76.3
5	94	88.3	83	78.6	75.2	73.3	71.9	71.2	70.7	70.4
6	92.8	85.9	79.6	74.3	70.2	68	66.3	65.5	64.9	64.4
7	91.6	83.6	76.2	70	65.3	62.7	60.6	59.7	59	58.5
8	90.3	81.3	72.8	65.7	60.3	57.3	55	53.9	53.2	52.6
9	89.1	78.9	69.4	61.4	55.4	52	49.4	48.2	47.3	46.7
10	87.9	76.6	66	57.1	50.4	46.6	43.8	42.4	41.5	40.7

The availability values obtained by the two objective functions 12 and 15 were compared to those determined by Buckland for loan periods of 1, 2, 5, and 10 weeks. Table 5 shows the percentage availability values computed by the proposed model for the current policy of seven books, with the corresponding Buckland (1975, p. 92) values for popularity class *B* one-copy loan policy.

It is not surprising to see that the second objective function yields the closest values to Buckland's. After all, the two models are based on the same assumption that users will always wait until the books are due back before returning them to the library. Naturally, complete agreement is not expected, since availability values calculated by the proposed model depend on the frequency distributions of user demands, and thus will vary from library to library. However, the similarity confirms that the assumptions made about availability are valid, and indicates that the model as a whole has a sound theoretical basis.

7. Conclusions

A new model for maximizing user satisfaction with a library's loan policy has been introduced. This integer programming model considers both elements of a loan policy: The loan period and the maximum number of books a user can borrow. User satisfaction is measured in terms of satisfaction with these two elements, as well as with book availability. By including these elements, the proposed model offers a more complete and realistic representation of the problem. The model has the advantage of easy solution, which can be obtained by simple spreadsheet software.

TABLE 5. Percentage availability values calculated by three methods.

	Weeks			
Loan period	1	2	5	10
Objective 12: 7 books	94	88	73	63
Objective 15: 7 books	94	89	72	44
Buckland (1975, p. 92)	94	86	62	44

A case study has been presented, illustrating how model parameters are calculated from user demand distributions. The values obtained compared well with previously published results. For future research, further investigation could be carried into the relative weights of the three measures of satisfaction mentioned above. Alternatively, loan policies for different groups, i.e., faculty, graduate and undergraduate students, could be simultaneously optimized. Furthermore, renewals and reservations (recalls), as well as duplication and multiple loan periods, could be included for a more comprehensive analysis.

References

Bookstein, A. (1975). Optimal loan periods. *Information Processing and Management, 11,* 235–242.

Bruce, D. R. (1975). A Markov model to study the loan dynamics at a reserve-loan desk at a lending library. *Library Quarterly, 45,* 161–179.

Buckland, M. K. (1972). An operational research study of a variable loan and duplication policy at the University of Lancaster. *Library Quarterly, 42,* 97–106.

Buckland, M. K. (1975). *Book availability and the library user*. New York: Pergamon Press.

Burkhalter, B. R., & Race, P. A. (1968). Analysis of renewals and overdues and other factors influencing the optimal charge-out period. In B. R. Burkhalter (Ed.), *Case studies in systems analysis in a university library* (pp. 11–33). Metuchen, NJ: Scarecrow.

Burrell, Q. L. (1980). A simple stochastic model for library loans. *Journal of Documentation, 36,* 115–132.

Burrell, Q. L., & Fenton, M. R. (1994). A model for library book circulations incorporating loan period. *Journal of the American Society for Information Science, 45,* 101–116.

Goehlert, R. (1978). Book availability and delivery service. *The Journal of Academic Librarianship, 4,* 368–371.

Goyal, S. K. (1970). Applications of operational research to the problem of determining appropriate loan periods for periodicals. *Libri, 20*(1), 94–100.

Hindle, A., & Buckland, M. K. (1976). Toward an adaptive loan and duplication policy for a university library. In P. Brophy, M. K. Buckland, & A. Hindle (Eds.), *Reader in operations research for libraries* (pp. 69–76). Englewood, CO: Indian Head.

Hindle, A., & Worthington, D. (1980). Simple stochastic models for library loans. *Journal of Documentation, 36,* 209–213.

Kantor, P. B. (1976a). Availability analysis. *Journal of the American Society for Information Science, 27,* 311–319.

Kantor, P. B. (1976b). The library as an information utility in the university context: Evolution and measurement of service. *Journal of the American Society for Information Science, 27,* 100–112.
Kantor, P. B. (1979). A review of library operations research. *Library Research, 1,* 295–345.
Kraft, D. H., & Boyce, B. R. (1991). *Operations research for libraries and information agencies.* San Diego, CA: Academic Press.
Morse, P. M. (1968). *Library effectiveness: A systems approach.* Cambridge, MA: MIT Press.
Morse, P. M. (1979). A queueing theory Bayesian model for the circulation of books in a library. *Operations Research, 27,* 673–716.
Saracevic, T., Shaw, W. M., Jr., Kantor, P. B. (1977). Causes and dynamics of user frustration in an academic library. *College & Research Libraries, 38,* 7–18.
Shaw, W. M., Jr. (1976a). Library-user interface: A simulation of the circulation subsystem. *Information Processing & Management, 12,* 77–91.
Shaw, W. M., Jr. (1976b). Loan period distribution in academic libraries. *Information Processing & Management, 12,* 157–159.
Sinha, B. K., & Clelland, R. C. (1976). Modeling for the management of library collections. *Management Science, 22,* 547–557.
Watters, L. J. (1967). Reduction of integer polynomial programming problems to zero-one linear programming. *Operations Research, 15,* 1171–1174.
Yagello, V. E., & Guthrie, G. (1975). The effect of reduced loan periods on high use items. *College and Research Libraries, 36,* 411–414.