# Integrated project operations and personnel scheduling with multiple labour classes 

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#### Abstract

The main theme of this paper is improving project schedules by integrating the scheduling of project jobs and labour resources. An ILP model is presented of the integrated project operations and personnel scheduling problem with multiple labour categories. Traditionally, this problem is solved in


two steps: first, operations are scheduled by solving the resource-constrained project scheduling problem; then, labour categories are scheduled by solving the personnel days-off scheduling problem. The proposed model combines the two stages into an integrated problem, which is solved in one step. Using 48 test problems, the two methods were compared in terms of total cost, labour cost and scheduling efficiency. The results clearly indicate that the integrated model outperforms the traditional two-step method.

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## 1. Introduction

The significant effect of scheduling on project labour productivity has been widely recognized in the literature, e.g. Oglesby et al. (1989), and Thomas and Napolitan (1995). Oglesby et al. (1989) emphasized the importance of project scheduling, stating that completeness, accuracy and timeliness of project schedules can considerably improve project productivity. Traditionally, project operations ( jobs) are scheduled first, fixing daily labour demands, then personnel are assigned to work shifts in order to satisfy these demands. As the two steps are interdependent, integrating them improves the efficiency of project schedules.

The major purpose of this paper is to present a model of the integrated project operations and personnel scheduling for multiple labour categories. First, the integer linear programming (ILP) model of the integrated mul-tiple-resource scheduling (IMRS) problem is developed. Then the ILP models of the traditional two-step method are presented for both steps: (i) resource-constrained project scheduling; and (ii) manpower days-off scheduling. Subsequently, the new model is compared to the traditional two-step approach in terms of the total cost, labour cost and labour utilization. Finally, an analysis is performed to determine the factors that affect the IMRS's improvement over the two-step method.

## 2. Literature review

A project can be defined as a large-scale system of sequenced operations (also called jobs). Traditionally, project operations are scheduled first to determine each job's duration and start time, given precedence restrictions and resource limitations. The critical path method (CPM) for project scheduling assumes unlimited resource availability, but in the real world resources are always limited.

Many project scheduling techniques have been developed to include resource considerations. Davis (1973) categorized project scheduling problems into three types: (i) time/cost trade-off problems; (2) resource levelling or smoothing; and (iii) resource-constrained scheduling. Icmeli et al. (1993) added a fourth category: the payment scheduling problem, in which the objective is to maximize the net present value of project cash flows. This paper is concerned with resource-constrained project scheduling, with limited availability of multiple labour resources.

Elmaghraby (1977), Hiroaka (1980), and Talbot (1982) presented very similar ILP formulations of the resource-constrained project scheduling problem. These models schedule project operations with time/cost trade-
off crashing opportunities. Dean et al. (1992) combined project scheduling with efficient allocation of labour demands. Recently, Alfares and Bailey (1997) presented a model of the integrated project task and labour scheduling for a single labour category. Of the many surveys of literature on resource-constrained project scheduling, the latest was compiled by Ozdamar and Ulusoy (1995).

Personnel scheduling is classified by Baker (1976) into three problems: (i) time-of-day or shift scheduling; (ii) day-of-week or days-off scheduling; and (iii) tour scheduling, which combines the first two. Tiberwala et al. (1972), Baker (1974), and Browne and Tiberwala (1975) formulated models in which only consecutive pairs of off days are allowed per week. Baker's (1976) ILP set covering model of personnel days-off scheduling allows the costs of different days-off shift patterns to vary, thus it can be used to minimize either the total number or total cost of workers assigned. Thompson (1990) included resource constraints in an LP model for shift scheduling with limited availability of employees. Tien and Kamiyama (1982), and Bedworth and Bailey (1987) provided comprehensive reviews of literature on labour scheduling.

## 3. Assumptions

The following assumptions apply both to the proposed IMRS model and to the traditional two-step model.
(1) As 1 day is the smallest time unit considered in both project scheduling and manpower days-off scheduling, it is the smallest time unit considered in this paper. Consequently, only a single (day) work shift is allowed per day.
(2) Only labour resources are considered in scheduling. However, the workforce is assumed to consist of multiple types of workers representing different crafts.
(3) Time and manpower requirements for each job (operation) are assumed to be given integer quantities, where the time is a decreasing function of manning level.
(4) Once a job is started on a given performance schedule (duration and associated start time), it must be completed without interruption on the same schedule.
(5) Only the seven weekly shift patterns with two consecutive off days per week are considered. As a weekend workday is paid at premium rate, the cost of each weekly shift pattern depends on the number of weekend workdays it contains.
(6) Once the project is started, the work on it continues uninterrupted, 7 days a week, until the project is completed.

## 4. The IMRS problem's ILP model

The following definitions are required to develop the IMRS model.

| $b_{\text {wid }}$ | $\begin{cases}1 & \text { if day } d \text { is a work-day for shift pattern } \\ 0 & \text { otherwise }\end{cases}$ |
| :---: | :---: |
| $C_{i k}$ | weekly cost per category $k$ worker on shift pattern $i$ |
| CP | project critical path length (minimum duration) in days |
| DD | project due date (maximum duration) in days |
| $E S_{j}$ | earliest start day of job (operation) $j$ |
| 7 | total number of jobs in the project |
| $\kappa$ | number of labour categories |
| L | set of the last jobs in the project which have no successors |
| $L S_{j t}$ | latest start day of job $j$ when its time (duration) is $t$, as determined by the due date $D D$ |
| $M_{j t k}$ | number of class $k$ workers required per day for job $j$ when its duration is $t$ days |
| OH | project overhead cost per day |
| $P_{j}$ | set of all jobs immediately preceding job $j$ |
| $T_{j}$ | set of all possible durations of job $j$ (in days) |
| W | $\begin{aligned} & \text { project due date in weeks }=(\text { smallest integer } \\ & \geq D D / 7) \end{aligned}$ |
| $W F_{k}$ | workforce size, i.e. total number of category $k$ workers available |

4.1. Decision variables
$P T \quad$ project duration in days, $C P \leq P T \leq D D$ and integer
$X_{j d t} \quad \begin{cases}1 & \text { if job } j \text { is started on day } d \text { with duration } t, \\ 0 & \text { otherwise }\end{cases}$ $j=1, \ldots, \mathcal{J}, E S_{j} \leq d \leq L S_{j t}, t \in T_{j}$
$r_{w i k} \quad$ number of category $k$ workers assigned to days-off shift pattern $i$ in week $w, r_{\text {wik }} \geq 0$ and integer, $w=1, \ldots, W, \quad i=1, \ldots, 7$, $k=1, \ldots, \kappa$

### 4.2. The objective function (minimizing total cost)

The objective sought is minimizing the total project cost, which is the sum of two components. The first com-
ponent is the total labour cost, which is the sum of weekly costs $C_{i k}$ of each days-off shift pattern $i$ for labour class $k$ times the number of workers assigned to it $Y_{\text {wik }}$ in every week $w$. The second component is the total overhead cost, which is the daily overhead cost $O H$ multiplied by project duration in days $P T$. Therefore, the objective function is written as

$$
\begin{equation*}
\text { Minimize } \sum_{w=1}^{W} \sum_{i=1}^{7} \sum_{k=1}^{K} C_{i k} * r_{w i k}+O H * P T \tag{1}
\end{equation*}
$$

The objective function is minimized subject to the following constraints.

### 4.3. Unique job performance constraints

Each job (operation) in the project must be performed once and only once. A unique performance schedule (duration and start time) must be chosen. Thus, only one variable $X_{j d t}$ associated with each job must equal one.

$$
\begin{equation*}
\sum_{d=E S_{j}}^{L S_{j i}} \sum_{u \in T_{j}} X_{j d t}=1, \quad j=1, \ldots, \mathcal{f} \tag{2}
\end{equation*}
$$

Elmaghraby (1977) proposed imposing unique job performance constraints (2) only on the last set of jobs with no successors $L$. The transitivity of the following precedence constraints (3), he explained, would force the remaining jobs to be uniquely performed. Although this approach reduces the number of ILP constraints, it increases solution time. Computational experience by Alfares (1991) showed that it is more efficient to restrict all the jobs in the project. Thus, unique performance constraints (2) are imposed on all jobs in the given project.

### 4.4. Precedence constraints

A job start day $d_{j}$ must be later than the completion day of any immediate predecessor $\left(d_{p}+t_{p}-1\right)$. Because the job can start, at the earliest, on the day following its predecessor's completion day $\left(d_{p}+t_{p}\right)$, this restriction is expressed as

$$
\begin{align*}
\sum_{d=E S_{p}}^{L S_{d t}} \sum_{\cup \in \mathcal{T}_{p}}(d+t) X_{p d t} \leq & \sum_{d=E S_{j}}^{L S_{j t}} \sum_{\cup t \in \mathcal{T}_{j}} d^{*} X_{j d t} \\
& \cup p \in P_{j}, j=1, \ldots, \mathcal{F} \tag{3}
\end{align*}
$$

For any job $j$, the number of precedence constraints (3) is equal to the number of immediate predecessors $\mathcal{N} P_{j}$. Hiroaka (1980) developed an alternative formulation in which only one precedence constraint is needed for each
job with predecessors, regardless of the number of immediate predecessors. Unlike constraints (3) above, Hiroaka's precedence constraints are not transitive, thus when they are used all the jobs in the project must be restricted by the unique performance constraints (2). Although Hiroaka's formulation uses fewer precedence constraints, Alfares's (1991) experience showed the above conventional formulation to be computationally more efficient.

### 4.5. Project completion constraints

Project duration in days $P T$ equals the latest job completion day $(d+t-1)$ among the set of last jobs without successors $L$. The constraints are written as

$$
\begin{equation*}
\sum_{d=E S_{j}}^{L S_{j t}} \sum_{\cup t \in T_{j}}(d+t-1) X_{j d t} \leq P T, \quad \cup j \in L \tag{4}
\end{equation*}
$$

### 4.6. Weekly labour (resource) constraints

In each week $w$, the total number of workers assigned for each category $k$ cannot exceed the size of the available workforce in that category $W F_{k}$. Because the availability of each resource (i.e. labour category) is limited, the following resource constraints must be included in the model

$$
\begin{equation*}
\sum_{i=1}^{7} r_{w i k} \leq W F_{k}, \quad w=1, \ldots, W, k=1, \ldots, \kappa \tag{5}
\end{equation*}
$$

### 4.7. Manpower scheduling constraints

For each day of the project's duration, the total number of workers assigned to each category must be greater than or equal to the sum of the required labour of all jobs that are active on that day. Assuming that a job $j$ is active on day $d$ if it has been started in day $q$ (i.e. completed on day $q+t-1$ ), then

$$
\begin{equation*}
q \leq d \leq q+t-1 \tag{6}
\end{equation*}
$$

rearranging,

$$
\begin{equation*}
d-t+1 \leq q \leq d \tag{7}
\end{equation*}
$$

But as $q$ is the job start day, the definition of $X_{j q t}$ dictates that

$$
\begin{equation*}
E S_{j} \leq q \leq L S_{j t} \tag{8}
\end{equation*}
$$

The combination of equations (7) and (8) yields

$$
\begin{equation*}
\max \left(d-t+1, E S_{j}\right) \leq q \leq \min \left(d, L S_{j t}\right) \tag{9}
\end{equation*}
$$

Because a job $j$ is active on day $d$ only if its start time $q$ satisfies equation (9), daily manpower scheduling constraints can be written as

$$
\begin{array}{r}
\sum_{j=1}^{\mathcal{L}} \sum_{U t \in T_{j}} \sum_{q=\max \left(d-t+1, E S_{j}\right)}^{\min \left(d, L S_{j t}\right)} M_{j t k} X_{j q t} \leq \sum_{w=1}^{W} \sum_{i=1}^{7} b_{w i d} r_{w i k} \\
k=1, \ldots, K, d=1, \ldots, D D \tag{10}
\end{array}
$$

The values of $b_{\text {wid }}$ correspond to the seven weekly shift patterns with two consecutive off days per week. Let shift pattern $i$ be off on days $(i, i+1)$ for $i=1, \ldots, 6$ and days $(7,1)$ for $i=7$, where day $1=$ Monday and day $7=$ Sunday.

## 5. The two-step method

Operations and labour scheduling for projects are usually performed in two separate steps. In the first step, jobs (operations) are scheduled using resource-constrained project scheduling techniques. This step determines job start times and durations, and thus the daily labour demands. In the second step, labour resources are scheduled to satisfy these demands, using manpower days-off scheduling techniques. This step determines the number of workers of each labour category assigned to each days-off shift pattern in every week. The ILP models of the two steps are presented below.

### 5.1. Step 1. Project operations scheduling

The objective at this stage is to minimize the cost of total labour man-day demands plus overheads. This is expressed as follows:

$$
\begin{equation*}
\text { Minimize } \sum_{k=1}^{K} L C_{k} \sum_{j=1}^{\mathcal{F}} \sum_{d=E S_{j}}^{L S_{i k}} \sum_{\cup \in T_{j}} t_{j} * M_{j t k} * X_{j d t}+O H * P T \tag{11}
\end{equation*}
$$

where: $L C_{k}=$ unit cost of class $k$ labour $=$ regular pay per non-weekend man-day.

Objective function (11) is subjected to constraints representing: unique job performance (2), precedence (3), and project completion (4). In addition, the following labour resource constraints must be included to ensure that daily labour demand does not exceed the workforce size for each labour category.

$$
\begin{align*}
\sum_{j=1}^{\mathcal{J}} \sum_{\cup t \in T_{j}} \sum_{q=\max \left(d-t+1, E S_{j}\right)}^{\min \left(d, L S_{j t}\right)} M_{j t k} X_{j q t} \leq W F_{k}, \\
k=1, \ldots, K, d=1, \ldots, D D \tag{12}
\end{align*}
$$

### 5.2. Step 2. Project labour scheduling

Having fixed the duration and start time of each operation, labour must now be assigned to perform these operations. What needs to be determined is the number of workers of different labour categories assigned to each of the seven days-off shift patterns in each week. Therefore, the following weekly days-off scheduling problem must be solved once for each week $w$ and every labour category $k$. The objective (13) is to minimize the weekly labour cost, subject to daily demand constraints (14).

$$
\begin{equation*}
\text { Minimize } \sum_{i=1}^{7} C_{i k} * r_{\text {wik }} \tag{13}
\end{equation*}
$$

subject to

$$
\begin{equation*}
\sum_{i=1}^{7} a_{i d} r_{w i k} \geq R_{d k}, \quad d=1, \ldots, 7 \tag{14}
\end{equation*}
$$

where

$$
\begin{aligned}
a_{i d}= & \begin{cases}1 & \text { if day } d \text { is a work-day for shift pattern } i \\
0 & \text { otherwise }\end{cases} \\
R_{d k}= & \text { total demand for labour of category } k
\end{aligned} \quad \text { on day } d=\text { left-hand side of constraint }(12) .20
$$

## 6. Computational results

The proposed integrated multiple-resouce scheduling (IMRS) model was compared with the traditional twostep model in terms of total cost, labour cost and labour utilization. A set of 48 test problems was carefully designed to allow the comparison over a variety of project types. The factors that affect the relative performance of the proposed scheduling procedure are naturally those with the greatest impact on the size of the ILP model, i.e. (i) number of jobs in the project $\mathcal{F}$; (ii) average number of predecessors per job $P$; (iii) average number of crashing opportunities $\bar{C}$, e.g. feasible staff-size/ job-duration combinations; and (iv) average job duration $\bar{T}$.

By allowing two values for each of the above four factors, $2^{4}=16$ combinations were possible. The number
of jobs 7 considered was either 6 or 8 , and the average number of predecessors $\bar{P}$ was either 0.6 or 1.0. Twelve (12) networks were constructed, shown in figure 1, such that a set of three networks was developed for each of the four combinations of $\bar{P}$ and $\mathcal{F}$. By varying job durations and crashing opportunities, each network in turn was used to generate four test problems. The problems involved either 1.5 or _ 2.0 for the average number of crashing opportunities $C$, and either 4 or 8 days for average job duration $T$.

To solve the 48 test problems, two programs were written in $\mathrm{C} / \mathrm{C}^{++}$programming language for any IBM or compatible DOS-based personal computer. One program used the proposed integrated solution method, while the other employed the two-step method. In either case, the $\mathrm{C} / \mathrm{C}^{++}$code was used to develop ILP coefficient matrices that represented the problems inside the EXCEL spreadsheet. LINDO's linear programming software spreadsheet version 'What's Best' was used to optimally solve the models. Table 1 summarizes the characteristics of the 48 test problems, and the percentage improvement in total cost, labour cost and labour utilization obtained by using the IMRS model over the two-step model.

### 6.1. Total project cost

Total project cost has been defined in equation (1) as the sum of labour and overhead costs. For the two-step method, the labour cost obtained by equation (15) is summed over all weeks and labour classes, then added to the overhead cost obtained from equation (13). In terms of total cost, the integrated method performed better than the two-step method in all but two cases, where it did equally well. The per cent savings produced by the integrated method ranged from $0.00 \%$ to $19.36 \%$, with an average of $5.65 \%$ and standard deviation of $4.46 \%$. Because the problem set was designed over a variety of test conditions, it is hard to argue that the data are normally distributed. However, the per cent savings of the integrated method are normally distributed and tests of significance are possible. The hypothesis that improvement in total cost is equal to zero was easily rejected at a significance level $\alpha$ of 0.01 .

### 6.2. Labour cost

The labour costs obtained by the integrated method were also lower than those produced by the two-step method in 46 problems. In the two remaining problems, the costs obtained by the two methods were identical.


Figure 1. (a) Problem networks with (a) $\mathcal{F}=6$ and $P=0.6$. (b) $\mathcal{F}=6$ and $P=1.0$. (c) $\mathcal{F}=8$ and $P=0.6$. (d) $\mathcal{F}=8$ and $P=1.0$.

The per cent savings in labour cost with respect to the two-step method ranged from 0.00 to $42.56 \%$, with an average of $12.71 \%$ and standard deviation of $8.81 \%$. The hypothesis that per cent improvement in labour cost is equal to zero was easily rejected at a significance level $\alpha$ of 0.01 .

### 6.3. Labour utilization

Labour utilization is a measure of scheduling efficiency. Because efficiency is the ratio of output (productive labour time) to input (total labour time), utilization is defined as:

Table 1. Performance improvement of the integrated method over the two-step method.

| Prob. no. | $\begin{aligned} & \text { Net. } \\ & \text { no. } \end{aligned}$ | f | $\bar{P}$ | C | T | Total cost \% cut | Labour cost \% cut | Utilization \% up |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 6 | 0.6 | 1.5 | 4 | 15.50 | 29.06 | 26.53 |
| 2 | 1 | 6 | 0.6 | 1.5 | 8 | 0.00 | 0.00 | 0.00 |
| 3 | 1 | 6 | 0.6 | 2 | 4 | 3.22 | 6.45 | 4.93 |
| 4 | 1 | 6 | 0.6 | 2 | 8 | 1.55 | 3.34 | 5.38 |
| 5 | 2 | 6 | 0.6 | 1.5 | 4 | 7.00 | 13.04 | 15.97 |
| 6 | 2 | 6 | 0.6 | 1.5 | 8 | 7.00 | 15.37 | 16.42 |
| 7 | 2 | 6 | 0.6 | 2 | 4 | 12.26 | 25.51 | 25.08 |
| 8 | 2 | 6 | 0.6 | 2 | 8 | 5.40 | 12.13 | 12.36 |
| 9 | 3 | 6 | 0.6 | 1.5 | 4 | 1.46 | 3.19 | 3.62 |
| 10 | 3 | 6 | 0.6 | 1.5 | 8 | 9.78 | 22.30 | 19.89 |
| 11 | 3 | 6 | 0.6 | 2 | 4 | 12.49 | 26.50 | 24.61 |
| 12 | 3 | 6 | 0.6 | 2 | 8 | 5.14 | 12.03 | 6.92 |
| 13 | 4 | 6 | 1 | 1.5 | 4 | 0.86 | 2.03 | 7.84 |
| 14 | 4 | 6 | 1 | 1.5 | 8 | 0.34 | 1.09 | 1.09 |
| 15 | 4 | 6 | 1 | 2 | 4 | 5.55 | 12.17 | 10.94 |
| 16 | 4 | 6 | 1 | 2 | 8 | 0.00 | 0.00 | 0.00 |
| 17 | 5 | 6 | 1 | 1.5 | 4 | 19.36 | 42.56 | 41.96 |
| 18 | 5 | 6 | 1 | 1.5 | 8 | 1.06 | 2.61 | 2.69 |
| 19 | 5 | 6 | 1 | 2 | 4 | 9.58 | 22.69 | 23.33 |
| 20 | 5 | 6 | 1 | 2 | 8 | 7.44 | 21.38 | 17.55 |
| 21 | 6 | 6 | 1 | 1.5 | 4 | 4.63 | 11.36 | 11.36 |
| 22 | 6 | 6 | 1 | 1.5 | 8 | 4.29 | 11.73 | 12.40 |
| 23 | 6 | 6 | 1 | 2 | 4 | 3.76 | 10.12 | 6.95 |
| 24 | 6 | 6 | 1 | 2 | 8 | 3.47 | 8.90 | 14.09 |
| 25 | 7 | 8 | 0.6 | 1.5 | 4 | 7.41 | 19.07 | 15.51 |
| 26 | 7 | 8 | 0.6 | 1.5 | 8 | 2.32 | 4.71 | 5.42 |
| 27 | 7 | 8 | 0.6 | 2 | 4 | 6.72 | 12.08 | 13.74 |
| 28 | 7 | 8 | 0.6 | 2 | 8 | 5.72 | 18.10 | 11.86 |
| 29 | 8 | 8 | 0.6 | 1.5 | 4 | 3.99 | 7.13 | 3.64 |
| 30 | 8 | 8 | 0.6 | 1.5 | 8 | 2.10 | 7.81 | 5.07 |
| 31 | 8 | 8 | 0.6 | 2 | 4 | 6.53 | 11.73 | 13.60 |
| 32 | 8 | 8 | 0.6 | 2 | 8 | 4.18 | 16.17 | 13.83 |
| 33 | 9 | 8 | 0.6 | 1.5 | 4 | 10.23 | 22.86 | 22.74 |
| 34 | 9 | 8 | 0.6 | 1.5 | 8 | 1.31 | 8.80 | 5.95 |
| 35 | 9 | 8 | 0.6 | 2 | 4 | 11.66 | 26.19 | 26.03 |
| 36 | 9 | 8 | 0.6 | 2 | 8 | 5.30 | 21.49 | 13.18 |
| 37 | 10 | 8 | 1 | 1.5 | 4 | 4.60 | 8.78 | 7.76 |
| 38 | 10 | 8 | 1 | 1.5 | 8 | 8.10 | 15.59 | 15.51 |
| 39 | 10 | 8 | 1 | 2 | 4 | 16.10 | 15.77 | 15.30 |
| 40 | 10 | 8 | 1 |  | 8 | 5.67 | 10.57 | 10.53 |
| 41 | 11 | 8 | 1 | 1.5 | 4 | 3.31 | 6.67 | 7.92 |
| 42 | 11 | 8 | 1 | 1.5 | 8 | 0.16 | 0.44 | 0.41 |
| 43 | 11 | 8 | 1 | 2 | 4 | 5.90 | 12.32 | 12.41 |
| 44 | 11 | 8 | 1 | 2 | 8 | 1.23 | 7.85 | 8.04 |
| 45 | 12 | 8 | 1 | 1.5 | 4 | 5.98 | 12.37 | 13.17 |
| 46 | 12 | 8 | 1 | 1.5 | 8 | 1.39 | 3.85 | 4.25 |
| 47 | 12 | 8 | 1 | 2 | 4 | 9.09 | 17.99 | 16.00 |
| 48 | 12 | 8 | 1 | 2 | 8 | 1.04 | 9.08 | 2.03 |

$$
\begin{equation*}
\text { Utilization }=\frac{\text { total required mandays }}{\text { total assigned mandays }} \tag{15}
\end{equation*}
$$

Labour utilization percentages obtained by the integrated method were higher than those produced by the two-step method in all but two problems. In these two
cases, the values obtained by the two methods were identical. The percent labour utilization of the two-step method ranged from $55.4 \%$ to $89.5 \%$, with an average of $79.2 \%$ and standard deviation of $8.7 \%$. For the integrated method, the per cent labour utilization ranged from $76.2 \%$ to $99.5 \%$, with an average of $93.2 \%$ and

Table 2. Summary of the $F$-statistics obtained from the ANOVA tests.

| Factor | Total <br> cost | Labour <br> cost | Labour <br> utilization |
| :---: | :---: | :---: | :---: |
| $Z$ |  |  | -21.24 |
| $\bar{P}$ |  | -54.83 | -21.90 |
| $\bar{C}$ | +15.32 | +42.78 | +15.92 |
| $\bar{T}$ | -73.40 | -186.59 | -254.13 |

standard deviation of $8.0 \%$. The integrated method outperformed the two-step method, on average, by $18.7 \%$. The standard deviation of the relative difference in labour utilization was $12.9 \%$. The hypothesis that per cent improvement in labour utilization is equal to zero was easily rejected at a significance level $\alpha$ of 0.01 .

### 6.4. Analysis of variance

The four factors considered in designing the test problems are: (i) the number of jobs in the project $\mathcal{F}$; (ii) the average number of predecessors per job $\bar{P}_{-}$; (iii) the average number of crashing opportunities $\vec{C}$; and (iv) the average job duration $T$. Each one of these four factors had two possible values, thus we had a $2^{4}$ experiment. Analysis of variance (ANOVA) was carried out in order to examine the effect of each factor on the performance of the proposed integrated method. Table 2 summarizes the results, showing the $F$-statistics that result from the ANOVA. The sign of the $F$-statistic indicates whether the correlation is positive or negative, while the magnitude corresponds to the relative_impact_of each factor.

Table 2 shows that factors $\bar{C}$ and $\bar{T}$ have the most significant effect on the relative performance of the integrated method, as compared to the two-step method. The performance of the integrated solution improves as factor $C$ increases, and deteriorates as factor $T$ increases. To a lesser degree, the performance relatively deteriorates as either factor $\mathcal{F}$ or $P$ increases.

## 7. Conclusions

A new ILP model integrating the scheduling of project operations and multiple labour resources has been presented. The integrated multi-resource scheduling (IMRS) model is able to produce significant savings when compared to the traditional two-step method. Currently, project operations are scheduled first, fixing daily labour demands, then workers are scheduled to satisfy these demands. By allowing the simultaneous scheduling of operations and labour categories, the proposed model minimizes the project labour cost. The
model achieves its objective by scheduling project operations in a way that results in the most efficient assignment of labour resources. By maximizing the efficiency of labour scheduling, the integrated method ultimately improves labour productivity.

Using 48 test problems, the integrated and two-step methods were compared in terms of three perfomance measures: total cost, labour cost and labour utilization. On average, the integrated method yielded $5.7 \%$ savings in total cost, $12.7 \%$ savings in labour cost, and $18.7 \%$ improvement in labour utilization. It can be concluded that integrated scheduling of operations and labour resources can lead to lower-cost and higher-productivity projects.

It appears worthwile to pursue several new, related research avenues in the future. First, the computational difficulty of the integrated problem could be reduced by using either heuristic rules, decomposition approaches, e.g. dynamic progrmming, or a combination of both. Another extension would be to include non-monetary considerations in the objective function. For example, employee satisfaction, schedule smoothness and robustness (i.e. relative insensitivity to unpredictable changes). Finally, stochastic elements could be introduced by assuming that time and labour requirements are probabilistic variables.

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