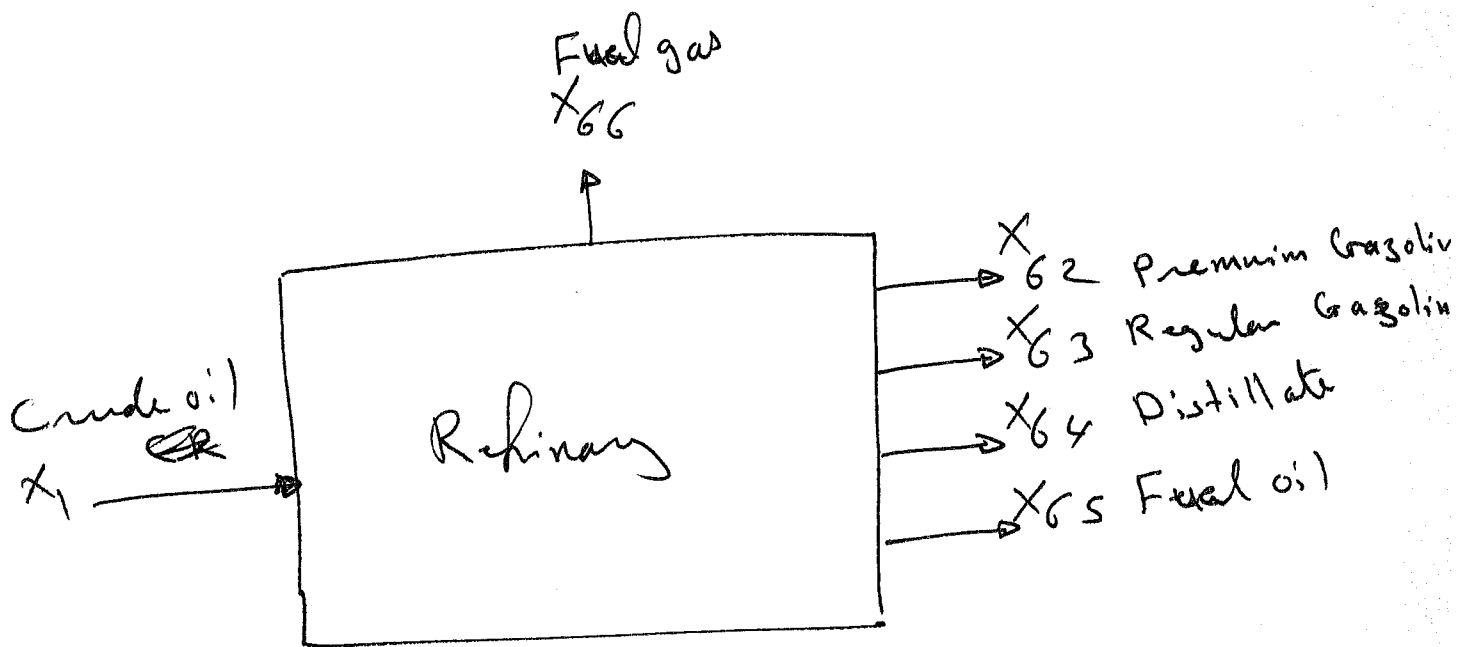


LINEAR PROGRAMMING IN REFINERY OPERATIONS

A PRACTICAL CASE STUDY

- The refinery for the LP model is shown in Figure 3.1
- The refinery is basically consist of :
 - An atmospheric distillation tower AD)
 - Reformer (RF)
 - Catalytic Cracker (CC)
 - Four blending tanks((PG,RG,DS,FO)
- The Mid Continent crude (stream 1) is fed to the AD tower



All LP models require plant data, and those are given in the text. For simplicity these costs etc are reproduced here:

- (1) the raw materials costs, unit operating costs and product selling prices are given in Table 3.1
- (2) the maximum capacities of the individual process units are given in Table 3.2
- (3) the yield patterns for the process units are given in Table 3.3
- (4) the product quality specifications are given in Table 3.4, and
- (5) the blending information from yields is given in Table 3.5.

The activity we shall be interested in is the flow of crude through the refinery. So, we shall define

XI as flow of a barrel of stream I per day
in units of 10^3 barrels/day.

Thus $XI = 1$ represents the flow of 1000 barrels/day. Further we shall be interested in carrying out many mass balances. Thus the fuel gas from the atmospheric tower is equal to 0.029 of the input crude to the tower for mid continent crude (see Table 3-3). As we are assuming the refinery is operating at steady state, the general mass balance.

Accumulation = Input - Output + Creation - Destruction
reduces to

$$0 = \text{Input} - \text{Output} + \text{Creation} - \text{Destruction}.$$

Now in the reformer and catalytic cracker we have both destruction and creation due to catalytic reactions. However these results are incorporated in the input output expressions through the use of yields in Table 3.3. Thus our mass balance becomes

$$0 = \text{Input} - \text{Output}$$

or $\text{Input} = \text{Output}$

For the fuel gas in the atmospheric tower, this is written as

$$.029 X1 = X3$$

This is a very restrictive equation for an LP program:

to overcome this restriction we generally write that the output in a mass balance is less than or equal to the input i.e.

$$\text{output} - \text{input} \leq 0$$

This is much more flexible for an LP program. Our fuel gas atmospheric tower balance then becomes

$$X3 - .029X1 \leq 0.$$

As we shall see this may result in the mass balance for the refinery not being complete, i.e. throwaway material will exist. This problem we shall discuss later. We are now ready to prepare our model. We shall employ the stream numbers given in Figure 3.1 for our model. Proceeding step by step:

- (1) availability of crude constraint.

we shall assume that 200,000 barrels of mid-continent crude are available for processing every day. This gives

$$X1 \leq 200$$

- (2) vessel capacity limitations.

these are given in Table 3.2 Using the appropriate input streams from Figure 3.1 gives

atmospheric tower: $X1 \leq 100$

Reformer: $X15 \leq 20$

Catalytic Cracker: $X21 + X27 \leq 30$

- (3) Vapour pressure quality specifications

The product quality specifications for both gasoline products is given in Table 3.4, and that for the various streams entering the blending tanks in Table 3.5. This gives for

Premium gasoline: $6.54X17 - X43 + 18.4X44 + 6.9X48 + 2.57X56 + 199.2X60 - 12.7X62 \leq 0$

Regular gasoline: $6.54X19 - X43 + 18.4X46 + 6.9X50 + 2.57X58 + 199.2X61 - 12.7X63 \leq 0.$

Note in both these equations we have introduced a penalty variable $X43$ which we shall incorporate in the objective function. The purpose of the penalty function is to ensure that the vapour pressure quality specification is adhered to as otherwise we may have to pay a penalty in selling our product. As a rule of thumb in all quality specifications, we should incorporate a penalty variable.

- (4) Density Quality Specifications

The density quality specification for the distillate and fuel oil products is given in Table 3-4 and for the various streams entering these blending tanks in Table 3-5. This gives for both products after incorporation of our penalty function.

Distillate: $295X23 + 292X29 + 272X31 - X43 + 294.4X52 - 306X64 \leq 0.$

Fuel Oil: $343X13 + 295X25 - X43 + 294.4X54 - 352X65 \leq 0.$

- (5) Sulfur Quality Specifications:

Data as for ^{sulfur} ~~density~~ specifications (Table 3-4 and Table 3-5)

$$\text{Distillate: } 0.98X_{23} + 0.526X_{29} + .283X_{31} - X_{43} + 0.353X_{52} - 0.5X_{64} \leq 0.0$$

$$\text{Fuel Oil : } 4.7X_{13} + 0.98X_{25} - X_{43} + 0.353X_{54} - 3.5X_{65} \leq 0.0$$

(6) Atmospheric Tower Balances:

Using the yields for mid-continent crude in Table 3.3 gives:

$$\begin{aligned}
 \text{Fuel gas:} & \quad - 0.09X_1 + X_3 \leq 0 \\
 \text{Straight Run Gasoline:} & \quad - 0.236X_1 + X_5 \leq 0 \\
 \text{Straight Run Maphtha:} & \quad - 0.223X_1 + X_7 \leq 0 \\
 \text{Straight Run Distallate} & \quad - 0.087X_1 + X_9 \leq 0 \\
 \text{Straight Run Fuel Oil:} & \quad - 0.111X_1 + X_{11} \leq 0 \\
 \text{Straight Run Residue:} & \quad - 0.315X_1 + X_{13} \leq 0
 \end{aligned}$$

(7) Component mass balances on the reformer using data for yields in Table 3.3 gives

$$\begin{aligned}
 \text{Fuel Gas} & \quad : - 0.129X_{15} + X_{33} \leq 0 \\
 \text{Reformer gasoline} & \quad : - 0.807X_{15} + X_{35} \leq 0
 \end{aligned}$$

(8) Component mass balances on the catalytic cracker using data for yields in table 3.3 gives

$$\begin{aligned}
 \text{Fuel Gas} & \quad : - .31X_{21} - .30X_{27} + X_{37} \leq 0 \\
 \text{Cat. Cracker Gasoline} & \quad : - .59X_{21} - .59X_{27} + X_{39} \leq 0 \\
 \text{Cat. Cracker Gas Oil} & \quad : - .22X_{21} - .21X_{27} + X_{41} \leq 0
 \end{aligned}$$

(9) Note mass balances on splitting functions

$$\begin{aligned}
 \text{Stream 5} & \quad : - X_5 + X_{44} + X_{46} \leq 0 \\
 \text{Stream 7} & \quad : - X_7 + X_{15} + X_{17} + X_{19} + X_{31} \leq 0 \\
 \text{Stream 9} & \quad : - X_9 + X_{27} + X_{29} \leq 0 \\
 \text{Stream 11} & \quad : - X_{11} + X_{21} + X_{23} + X_{25} \leq 0 \\
 \text{Stream 35} & \quad : - X_{35} + X_{56} + X_{58} \leq 0 \\
 \text{Stream 39} & \quad : - X_{39} + X_{48} + X_{50} \leq 0 \\
 \text{Stream 41} & \quad : - X_{41} + X_{52} + X_{54} \leq 0
 \end{aligned}$$

(10) Blending Balances for Products

$$\begin{aligned}
 \text{Premium gasoline:} & \quad - X_{17} - X_{44} - X_{48} - X_{56} - X_{60} + X_{62} \leq 0 \\
 \text{Regular gasoline:} & \quad - X_{19} - X_{46} - X_{50} - X_{61} + X_{63} \leq 0 \\
 \text{Distillate} & \quad : - X_{23} - X_{29} - X_{31} - X_{52} + X_{64} \leq 0 \\
 \text{Fuel Oil} & \quad : - X_{13} - X_{25} - X_{54} + X_{65} \leq 0
 \end{aligned}$$

(11) Operating cost equation for process units.

The operating costs of the atmospheric tower, catalytic reformer and catalytic - cracker is given in Table 3.1 per barrel processed. The total operating cost is set

equal to variable X68.

$$-0.1 \times X1 - 0.15 \times X15 - 0.08 \times X21 - 0.08 \times X27 + X68 = 0$$

(12) Total Butane consumed.

The total butane added to the blending tanks is given by variable X67

$$- X60 - X61 + X67 = 0.$$

(13) Fuel gas product.

The total fuel gas from the three operating vessels - tower, reformer and cracker - is summed and set equal to variable X66.

$$- X3 - X33 - X37 + X66 = 0.$$

(14) Gasoline product mass balance.

The total gasoline produced is premium or regular is summed and set equal to X70.

$$- X62 - X63 + X70 = 0.$$

(15) Distillate/Fuel oil mass balance

the total distillate/fuel oil produced is summed and set equal to variable X71

$$- X64 - X65 + X71 = 0.$$

(16) Octane number quality specifications.

The regular gasoline must have an octane number ≥ 85 and the premium gasoline must be greater than 90 as given in Table 3.4. The octane number for each of the streams added to the blending tanks for these gasoline is given in Table 3.5. A penalty cost function X69 is introduced to these equations to ensure compliance with these specifications.

$$\begin{aligned} \text{Premium Gasoline : } & 65. \times X17 + 78.5 \times X44 + 93.7 \times X48 \\ & + 104 \times X56 + 91.8 \times X60 - 90 \times X62 \\ & + X69 \geq 0 \end{aligned}$$

$$\begin{aligned} \text{Regular Gasoline : } & 65. \times X19 + 78.5 \times X46 + 93.7 \times X50 \\ & + 91.8 \times X51 - 85. \times X63 + X69 \geq 0 \end{aligned}$$

(17) Product Demand Requirements.

Management may specify how much of each product is to be produced. Initially these variables are just set greater than or equal to zero.

$$\text{Premium gasoline: } X62 \geq 0$$

Regular gasoline: X 63 \geq 0
Distillate : X 64 \geq 0
Fuel oil : X 65 \geq 0

This completes our simple one crude model for the refinery. The variables employed are summarized in Table 3.6 and the constraint rows in Table 3.7.

All that remains is to specify the objective function, which is to maximize the profit in the refinery. The raw material, operating costs and product prices are given in Table 3.1. Using the appropriate variables the objective function in 10^3 \$/day is

Maximize : $- 7.5*X1 - 100*X43 + 10.5*X62 + 9.1*X63 + 7.7*X64$
 $+6.65*X65 + 1.5*X66 - 6.75*X67 - 1.0*X68 - 100*X69$

Note that the two penalty function X43 and X69 have been given an extremely large coefficient so that the cost of breaking the constraints involving these functions is prohibitive.

The next step is to prepare our LP program to solve this model.

Table 3.7
Row Designation

<u>Row No.</u>	<u>Description</u>
1	Crude availability
2	Atmospheric Tower Capacity Limitation
3	Reformer capacity limitation
4	Cat. cracker capacity limitation
5	Premium gasoline Vapour Pressure quality Specification
6	Regular gasoline vapour pressure quality specification
7	Distillate density quality specification
8	Fuel oil density quality specification
9	Distillate sulfur quality specification
10	Fuel oil quality specification
11	Tower fuel gas balance
12	Tower straight run gasoline balance
13	Tower straight run naphtha balance
14	Tower straight run distillate balance
15	Tower straight run fuel oil balance
16	Tower straight run residue balance
17	Reformer fuel gas balance
18	Reformer gasoline balance
19	Cat. cracker fuel gas balance
20	Cat. cracker gasoline balance
21	Cat. cracker gas oil balance
22	Stream 5 node splitter balance
23	Stream 7 node splitter balance
24	Stream 9 node splitter balance

- 25 Stream 11 node splitter balance
- 26 Stream 35 node splitter balance
- 27 Stream 39 node splitter balance
- 28 Stream 41 node splitter balance
- 29 Premium gasoline blending
- 30 Regular gasoline blending
- 31 Distillate blending
- 32 Fuel Oil blending
- 33 Operating cost equation for process units
- 34 Butane consumption balance
- 35 Fuel gas product balance
- 36 Gasoline product total produced
- 37 Distillate/Fuel Oil total produced
- 38 Premium gasoline octane quality specification
- 39 Regular gasoline octane quality specification
- 40 Premium gasoline product demand
- 41 Regular gasoline product demand
- 42 Distillate product demand
- 43 Fuel oil product demand

LP model for one crude refinery

We have written this model using the IMSL library and using LINDO. For IMSL we have used the LP/FROTRAN version as it uses mnemonic names and is very simple to apply. Typical programs and specifications are supplied in the next nine pages.

The program for the one crude feed to the refinery is given in Table 3.8 together with its output. The result is identical to that given in the text for the two crude feed to the refinery in which only the mid continent crude is selected as a desirable feed.

In this brief write up, I have indicated how to prepare a simple LP model for a refinery. The preparation of the more complex model given in the text follows quite logically from the simple model derivation.

EXAMPLE #1:

$$\begin{aligned} \text{MAX} \quad & - 7.5 X1 - 100 X43 + 10.5 X62 + 9.1 X63 + 7.7 X64 \\ & + 6.65 X65 + 1.5 X66 - 6.75 X67 - X68 - 100 X69 \end{aligned}$$

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SUBJECT TO

- 2) $X1 \leq 100$
- 3) $X15 \leq 20$
- 4) $X21 + X27 \leq 30$
- 5) $- X43 - 12.7 X62 + 6.54 X17 + 18.4 X44 + 6.9 X48 + 2.57 X56 + 199.2 X60 \leq 0$
- 6) $- X43 - 12.7 X63 + 6.54 X19 + 18.4 X46 + 6.9 X50 + 2.57 X58 + 199.2 X61 \leq 0$
- 7) $- X43 - 306 X64 + 295 X23 + 292 X29 + 272 X31 + 294.39999 X52 \leq 0$
- 8) $- X43 - 352 X65 + 343 X13 + 295 X25 + 294.39999 X54 \leq 0$
- 9) $- X43 - 0.5 X64 + 0.98 X23 + 0.526 X29 + 0.283 X31 + 0.353 X52 \leq 0$
- 10) $- X43 - 3.5 X65 + 4.7 X13 + 0.98 X25 + 0.353 X54 \leq 0$
- 11) $- 0.029 X1 + X3 \leq 0$
- 12) $- 0.236 X1 + X5 \leq 0$
- 13) $- 0.111 X1 + X11 \leq 0$
- 14) $- 0.315 X1 + X13 \leq 0$
- 15) $- 0.807 X15 + X35 \leq 0$
- 16) $- 0.31 X21 - 0.3 X27 + X37 \leq 0$
- 17) $- 0.59 X21 - 0.59 X27 + X39 \leq 0$
- 18) $- 0.22 X21 - 0.21 X27 + X41 \leq 0$
- 19) $X44 + X46 - X5 \leq 0$
- 20) $X15 + X17 + X19 + X31 - X7 \leq 0$
- 21) $X27 + X29 - X9 \leq 0$
- 22) $X21 + X23 + X25 - X11 \leq 0$
- 23) $X56 + X58 - X35 \leq 0$
- 24) $X48 + X50 - X39 \leq 0$
- 25) $X52 + X54 - X41 \leq 0$
- 26) $X64 - X23 - X29 - X31 - X52 \leq 0$
- 27) $X65 - X13 - X25 - X54 \leq 0$
- 28) $- 0.1 X1 + X68 - 0.15 X15 - 0.08 X21 - 0.08 X27 = 0$
- 29) $X66 - X3 - X33 - X37 = 0$
- 30) $- X62 - X63 + X70 = 0$
- 31) $- X64 - X65 + X71 = 0$
- 32) $- 90 X62 + X69 + 65 X17 + 78.5 X44 + 93.7 X48 + 104 X56 + 91.8 X60 \geq 0$
- 33) $- 86 X63 + X69 + 65 X19 + 78.5 X46 + 93.7 X50 + 104 X58 + 91.8 X61 \geq 0$
- 34) $- 0.223 X1 + X7 \leq 0$
- 35) $- 0.087 X1 + X9 \leq 0$
- 36) $- 0.129 X15 + X33 \leq 0$
- 37) $X52 + X54 - X41 \leq 0$
- 38) $X64 - X23 - X29 - X31 - X52 \leq 0$
- 39) $X67 - X60 - X61 = 0$
- 40) $X62 - X17 - X44 - X48 - X56 - X60 \leq 0$
- 41) $X63 - X19 - X46 - X50 - X58 - X61 \leq 0$

END

SOLUTION TO PROBLEM #1

LP OPTIMUM FOUND AT STEP 44

OBJECTIVE FUNCTION VALUE

1) 12.283700

VARIABLE	VALUE	REDUCED COST
X1	89.717770	.000000
X43	.000000	94.421590
X62	42.298670	.000000
X63	.000000	.000000
X64	.000000	.000000
X65	36.809690	.000000
X66	7.523449	.000000
X67	.372995	.000000
X68	12.596210	.000000
X69	.000000	99.821530
X15	20.000000	.000000
X21	.000000	8.510704
X27	7.805446	.000000
X17	.007065	.000000
X44	21.173400	.000000
X48	4.605213	.000000
X56	16.140000	.000000
X60	.372995	.000000
X19	.000000	1.943945
X46	.000000	1.375423
X50	.000000	.855129
X58	.000000	.485868
X61	.000000	.000000
X23	.000000	12.915000
X29	.000000	4.148402
X31	.000000	1.170686
X52	.000000	16.389630
X13	25.211870	.000000
X25	9.958673	.000000
X54	1.639144	.000000
X3	2.601815	.000000
X5	21.173400	.000000
X9	7.805446	.000000
X11	9.958673	.000000
X33	2.580000	.000000
X35	16.140000	.000000
X37	2.341634	.000000
X39	4.605213	.000000
X41	1.639144	.000000
X7	20.007060	.000000
X70	42.298670	.000000
X71	36.809690	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	10.282220	.000000
3)	.000000	.610418
4)	22.194550	.000000
5)	.000000	.020786
6)	.000000	.015965
7)	.000000	.000000
8)	888.965900	.000000
9)	.000000	.000000
10)	.000000	5.541668
11)	.000000	1.500000
12)	.000000	9.573138
13)	.000000	20.615000
14)	3.049227	.000000
15)	.000000	11.694680
16)	.000000	1.500000
17)	.000000	10.880640
18)	.000000	24.089630
19)	.000000	9.573138
20)	.000000	8.870686
21)	.000000	11.848400
22)	.000000	20.615000
23)	.000000	11.694680
24)	.000000	10.880640
25)	.000000	24.089630
26)	.000000	7.700000
27)	.000000	26.045840
28)	.000000	-1.000000
29)	.000000	1.500000
30)	.000000	.000000
31)	.000000	.000000
32)	.000000	-.070294
33)	.000000	-.108172
34)	.000000	8.870686
35)	.000000	11.848400
36)	.000000	1.500000
37)	.000000	.000000
38)	.000000	.000000
39)	.000000	-6.750000
40)	.000000	4.437505
41)	.000000	.000000

discarded amount

NO. ITERATIONS= 44

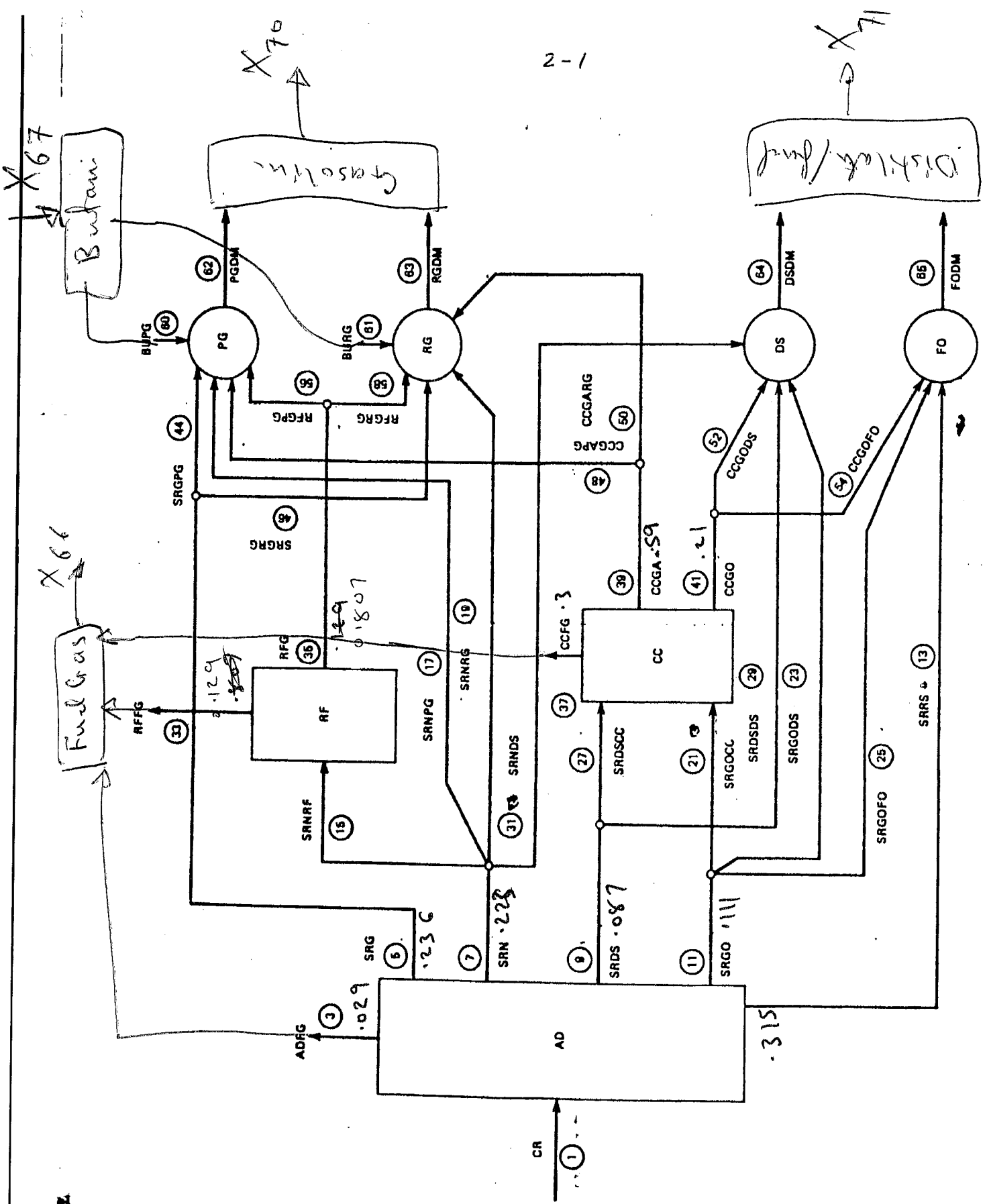


TABLE 3-1 Refinery Costs, Prices, and Symbols Used in Model

Raw material costs (\$/bbl)	
Mid Continent crude, CRM	7.50
West Texas crude, CRW	6.50
Butane, BU	6.75
Operating costs (\$/bbl processed)	
Atmospheric (crude) distillation, AD	0.10
Catalytic reformer, RF	0.15
Catalytic cracker, CC	0.08
Hydrotreater, HT	0.10
Product prices (\$/bbl)	
Premium gasoline, PG	10.50
Regular gasoline, RG	9.10
Distillate, DS	7.70
Heavy fuel oil, FO	6.65
Fuel gas, FG	1.50

**TABLE 3-2 Capacities of Process Units
(10³ bb/day)**

Atmospheric (crude) distillation unit	100
Catalytic reformer	20
Catalytic cracker	30

TABLE 3-3 Yield Patterns for Process Units

Output Yields		Mid Continent		West Texas	
Crude Unit (AD)		CR ①		CR ②	
③, ④	Fuel gas (FG)	.029		.017	
⑤, ⑥	Straight run gasoline (SRG)	.236		.180	
⑦, ⑧	Naptha (N)	.223		.196	
⑨, ⑩	Distillate (DS)	.087		.073	
⑪, ⑫	Gas oil (GO)	.111		.091	
⑬, ⑭	Residuum (RS)	.315		.443	
		1.000		1.000	
Reformer (RF)		SR ⑬		SR ⑭	
⑮, ⑯	Fuel gas (FG)	.129		.099	
⑰, ⑱	Reformate (G) (for gasoline blending)	.807		.836	
	Loss	.064		.065	
		1.000		1.000	
Catalytic Cracker (CC)		DS ⑲	GO ⑳	DS ㉑	GO ㉒
㉓, ㉔	Fuel gas (FG)	.30	.31	.36	.38
㉕, ㉖	Cracked gasoline (GA)	.59	.59	.58	.60
㉗, ㉘	Gas oil (GO) (for distillate or fuel oil)	.21	.22	.15	.15
		1.10	1.12	1.09	1.13
(GAIN)		(.10)	.12	.09	.13
Hydrotreater (HT)		RS ㉙			
㉚	Desulfurized residuum (RS)	.97			

TABLE 3-4 Product Quality Specifications

	Clear Research Octane	Vapor Pressure (mm Hg)	Density (lb/bbl)	Sulfur (lb/bbl)
Premium gasoline	(\geq) 90	(\leq) 12.7		
Regular gasoline	(\geq) 86	(\leq) 12.7		
Distillate			(\leq) 306	(\leq) 0.5
Heavy fuel oil			(\leq) 352	(\leq) 3.5

TABLE 3-5 Blending Information from Yields

Premium and Regular Gasoline	Clear Research Octane		Vapor Pressure (mm Hg)	
	Mid Continent	West Texas	Mid Continent	West Texas
Straight run gasoline (from crude unit)	78.5		18.4	
Naphtha (from crude unit)	65.0		6.54	
Reformate (from reformer)	104.0		2.57	
Cracked gasoline (from cat cracker)	93.7		6.90	
Butane	91.8		199.2	
	Density (lb/bbl)		Sulfur (lb/bbl)	
	Mid Continent	West Texas	Mid Continent	West Texas
<i>Distillate</i>				
Naphtha (from crude unit)	272.0	272.0	.283	1.48
Distillate (from crude unit)	292.0	297.6	.526	2.83
Gas oil (from crude unit)	295.0	303.3	.980	5.05
Gas oil (from cat cracker)	294.4	299.1	.353	1.31
<i>Heavy Fuel Oil</i>				
Gas oil (from cat cracker)	294.4	299.1	0.353	1.31
Gas oil (from crude unit)	295.0	303.3	0.980	5.05
Residuum (from crude unit)	343.0	365.0	4.700	11.00
Hydrotreated residuum (from hydrotreater)		365.0		6.00

Table 3.6
Summary of Variables

No.	Description	Mnemonic
X1	Tower crude feed	CR
X3	Tower fuel gas overheads	ADFG†
X5	Tower straight run gasoline	SRG
X7	Tower straight run naphtha	SRN
X9	Tower straight run distillate	SRDS
X11	Tower straight run gas oil	SRGO
X13	Tower straight run residuum	SRRS
X15	Straight run naphtha feed to reformer	SRNRF
X17	Straight run naphtha to PG blending	SRNPG
X19	Straight run naphtha to RG blending	SRNRG
X21	Straight run gas oil to cat. cracker	SRGOCC
X23	Straight run gas oil to distillate blend	SRGODS
X25	Straight run gas oil to fuel oil blend	SRGOFO
X27	Straight run distillate to cat. cracker	SRDSCC
X29	Straight run distillate to distillate Blend	SRDSDS
X31	Straight run naphtha to distillate blend	SRNDS
X33	Reformer fuel gas	RFFG
X35	Reformer gasoline	RFGO
X37	Cat. cracker fuel gas	CCFG
X39	Cat. cracker gasoline	CCGA
X41	Cat. cracker gas oil	CCGO
X43	Cost Penalty Function	-
X44	Straight run gasoline to premium gasoline blend	SRGPG
X46	Straight run gasoline to regular gasoline blend	SRGRG
X48	Cat. cracker gasoline to premium gasoline blend	CCGAPG

X50	Cat. cracker gasoline to Regular gasoline blend	CCGARG
X52	Cat. cracker gas oil to distillate blend	CCGODS
X54	Cat. cracker gas oil to fuel oil blend	CCGOFO
X56	Reformer gasoline to premium gasoline blend	RFGPG
X58	Reformer gasoline to regular gasoline blend	RFGRG
X60	Butane feed to premium gasoline blend	BUFG
X61	Butane feed to regular gasoline blend	BURG
X62	Premium gasoline product	PGDM
X63	Regular gasoline product	RGDM*
X64	Distillate product	DSDM
X65	Fuel Oil Product	FODM
X66	Fuel gas product	-
X67	Total Butane feed	-
X68	Total operating costs of equipment units	-
X69	penalty cost for breaking octane number specification	-
X70	Total gasoline product	-
X71	Total Fuel Oil/Distillate Product	-