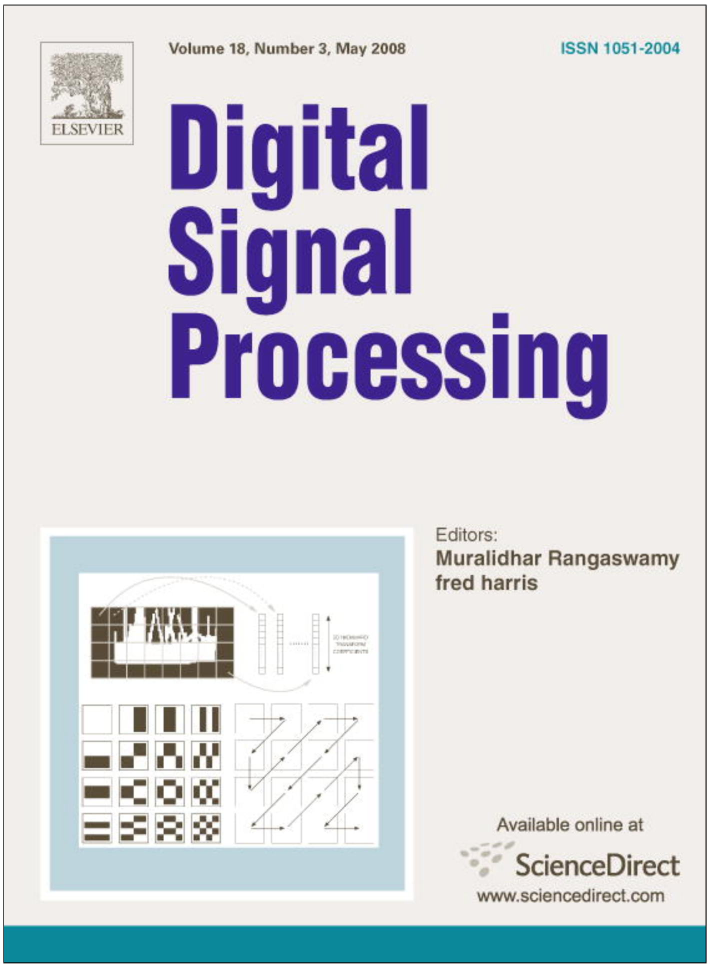
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Logarithmic quantization in the least mean squares algorithm

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Abstract

In this paper, we introduce a framework for adaptive filtering techniques with simplified recursion. The simplification is mainly carried out by rounding the full-precision error information of the recursion to their closest power-of-two values. A new method for power-of-two quantization is proposed in this study. The method uses companded delta modulation structure to perform the quantization. The proposed structure shows a performance that is comparable to that of full precision adaptive filters. Convergence analysis of this structure is included and closed-form expressions for the error statistics are derived. Furthermore, an efficient method for implementing the new structure is presented where only simple shift and loop operations are required.

Keywords: Adaptive filtering; Efficient algorithms; Least mean squares; Sign algorithm; Log-LMS; Delta modulation; Convergence analysis; Implementation; Power-of-two quantization

1. Introduction

Adaptive filtering techniques have gained increasing attention in many industrial applications. Some examples of these applications include channel estimation and equalization in communication systems, echo cancelation, and system identification. The least mean squares (LMS) algorithm is a widely used and well-established adaptive filtering algorithm. It is used to estimate parameters or weights from a measured process. The weights update of LMS is given by the recursion [1]

$$\boldsymbol{w}_i = \boldsymbol{w}_{i-1} + \mu \boldsymbol{u}_i^* e(i), \quad i \geqslant 0, \quad \boldsymbol{w}_0 = \text{initial weight vector}, \tag{1}$$

where $\boldsymbol{w}_i$ is the $M \times 1$ weight vector at iteration i, $\boldsymbol{u}_i$ is the $1 \times M$ input regression vector, $d(i)$ is the scalar desired signal, μ is the fixed adaptation step-size, and the superscript $(^*)$ indicates the conjugate transpose. The constant M is the number of taps for the LMS filter. Without loss of generality and for ease of analysis, the variables in this study are assumed to be real-valued. The a priori estimation error $e(i)$ is defined as [1]

$$e(i) \triangleq d(i) - \boldsymbol{u}_i \boldsymbol{w}_{i-1}, \tag{2}$$

where $d(i)$ is the desired signal. The mean-square error (MSE) is defined here as

$$\text{MSE} \triangleq E\left|e(i)\right|^2, \tag{3}$$

where E denotes the expected value of its argument.

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The performance of adaptive filtering techniques is usually measured in terms of the final mean square error and convergence speed. In the LMS algorithm, these quantities are controlled mainly by the adaptation step-size μ. Increasing the step-size would improve the convergence speed but at the same time would increase the final MSE [2]. Generally, an optimum step-size trend is sought that results in the best trade-off between convergence speed and final MSE.

The complexity of adaptive filters is usually measured by the number of operations (multiplications and additions) performed by the filter during the learning phase. New applications strive for both reduced operation count and increased convergence speed of adaptive filters. This is especially important for high speed applications where hardware implementation is necessary [3].

Some algorithms, such as recursive least squares (RLS), improve the convergence speed over LMS at the expense of increased complexity [4]. On the other hand, the sign and sign–sign LMS algorithms decrease the complexity by eliminating the need for real multiplication inside the adaptation recursion. However, these algorithms usually offer either poor convergence speed or large final MSE.

There have been many attempts in literature to reduce the complexity of adaptive filters without compromising performance. One of these attempts is to quantize the error signal in the weight recursion to its nearest power-of-two value [5–9]. In other words,

$$\boldsymbol{w}_i = \boldsymbol{w}_{i-1} + \mu \boldsymbol{u}_i^* Q_{\log}\big(e(i)\big), \tag{4}$$

where the logarithmic quantizer $Q_{\log}(.)$ simply rounds its input to the nearest power-of-two value. This quantizer is refered to in short as *log-quantizer* (or, equivalently, power-of-two quantizer) while the recursion algorithm is called log-LMS. The binary representation of the term $Q_{\log}(e(i))$ has a single digit that is “1” and the rest are zeros. Therefore the multiplication of $e(i)$ by the regression vector in the original LMS becomes now a simple shift operation. Provided that the step-size μ is chosen as power-of-two, it is obvious that the recursion (4) is free of any real multiplications. This in turn can drastically reduce the complexity of the LMS algorithm especially when the order of the adaptive filter is high.

The log-LMS algorithm described by (4) can be represented in a block diagram as shown in Fig. 1. The heavy lines indicate vector operations while the thin lines indicate scalar operations. The a priori error $e(i)$ is computed by subtracting the quantity $\hat{d}(i) = \boldsymbol{u}_i \boldsymbol{w}_{i-1}$ from the desired signal $d(i)$. The a priori error is then passed through the log-quantizer which rounds it to its nearest power-of-two value. This value is then multiplied by the step-size μ and the regression vector $\boldsymbol{u}_i^*$ and finally accumulated to result in the updated weight vector $\boldsymbol{w}_i$.

An obvious way of obtaining a power-of-two quantization for real inputs is given by [7]

$$Q^1_{\log}(x) = 2^{\lfloor \log_2 |x| \rfloor}\,\mathrm{sign}(x), \tag{5}$$

where $\lfloor z \rfloor$ is the largest integer less than z. One of the disadvantages of this quantizer is that it has infinite number of bits while in practice, the number of bits of this quantizer is usually finite. This restriction was relaxed in [5] by introducing a finite-bit log-quantizer where the quantization operation is defined by

$$Q^2_{\log}(x) = \begin{cases} \mathrm{sign}(x), & |x| \geqslant 1, \\ 2^{\lfloor \log_2 |x| \rfloor}\,\mathrm{sign}(x), & 2^{-B+1} \leqslant |x| < 1, \\ 0, & |x| < 2^{-B+1}, \end{cases} \tag{6}$$

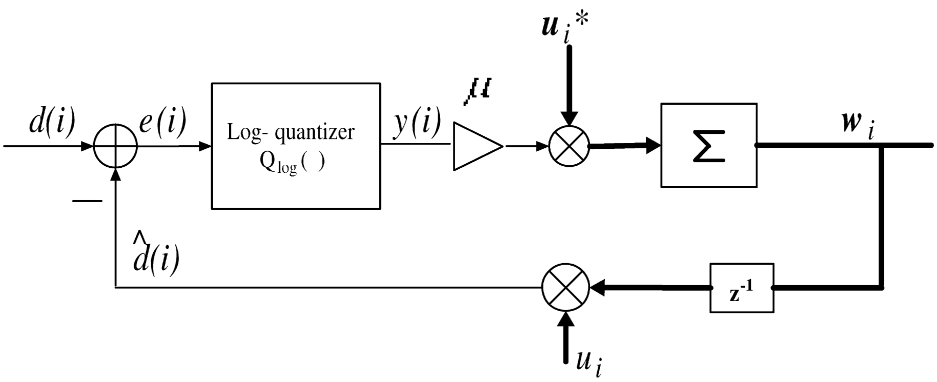

Fig. 1. Block diagram representation of the modified LMS algorithm with logarithmic quantization.

where B is the number of bits assigned to this quantizer. The analysis included in [5] are restricted to the case where the covariance matrix $\mathbf{R} = E\{u_i^* u_i\}$ is diagonal. This algorithm stops adaptation when the magnitude of the error falls below the *dead zone* region 2^{-B+1}. This problem was resolved in [10] by modifying the quantizer as follows:

$$Q_{\log}^3(x) = \begin{cases} \operatorname{sign}(x), & |x| \geqslant 1, \\ 2^{\lfloor \log_2 |x| \rfloor} \operatorname{sign}(x), & 2^{-B+1} \leqslant |x| < 1, \\ 2^{-B+1} \operatorname{sign}(x), & |x| < 2^{-B+1}. \end{cases} \tag{7}$$

An expression for the steady-state excess error of this algorithm was derived in [10] without using the assumption that $\mathbf{R}$ is diagonal.

While in these methods, only the error signal is quantized, there are other studies that applied logarithmic quantization to both the error and the regressor vector. One example is the so-called log–log LMS algorithm investigated in [6]. In this case, the update recursion is given by

$$\boldsymbol{w}_i = \boldsymbol{w}_{i-1} + \mu Q_{\log}\big(\boldsymbol{u}_i^*\big) Q_{\log}\big(e(i)\big). \tag{8}$$

This method further reduces the complexity of the LMS algorithm by eliminating even the shift operation. While the work in [6] discussed the computation complexity of the log–log LMS, it did not give any performance analysis of this algorithm. Moreover, it has been shown in [7] that the quantization of the regression vector may cause instability of the algorithm.

In this work, we propose an alternative method for logarithmic quantization of the error signal. The new method uses a companded single-bit delta modulator (DM) to perform the quantization. Furthermore, a novel approach for analyzing this algorithm is presented. The quantizer is shown to be equivalent to a random gain with known statistics. This, in turn, helps in coming up with closed-form expressions for the error statistics of this algorithm. Moreover, a simple approach for implementing the proposed quantizer is presented. Finally, simulations are used to verify the analytical results of the proposed algorithm and also to show the superior performance of this algorithm.

This paper is organized as follows. The proposed algorithm is described in Section 2. Analysis of this algorithm is included in Section 3. In Section 4, the implementation of the proposed log-quantizer is considered. In this section, we show that this quantizer can be implemented using simple shift and loop operations. In Section 5, the performance of the proposed algorithm is investigated via simulations and compared with that of conventional adaptive filtering algorithms.

2. Proposed DM-based log-LMS algorithm

The proposed logarithmic quantizer is shown in Fig. 2. This structure is in fact a *companded* delta modulator. (A linear delta modulator in-between logarithmic and exponential functions.) The motivation behind the use of this structure is twofold. First, this structure was first introduced in [11] and it was shown that it has excellent tracking and dynamic range performance [12,13]. Second, this structure can be implemented through simple shift and loop operations, as will be shown in the implementation section of this work.

The linear delta modulator is shown in Fig. 3. Figure 3a shows the single-bit structure while Fig. 3b shows the approximation of the quantizer by an additive noise $e_Q(i)$. Delta modulators track their input by means of incrementing

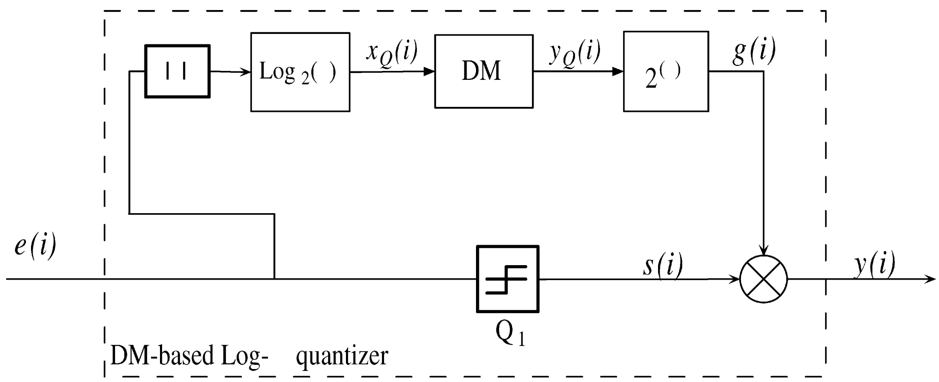

Fig. 2. A proposed DM-based log-quantizer.

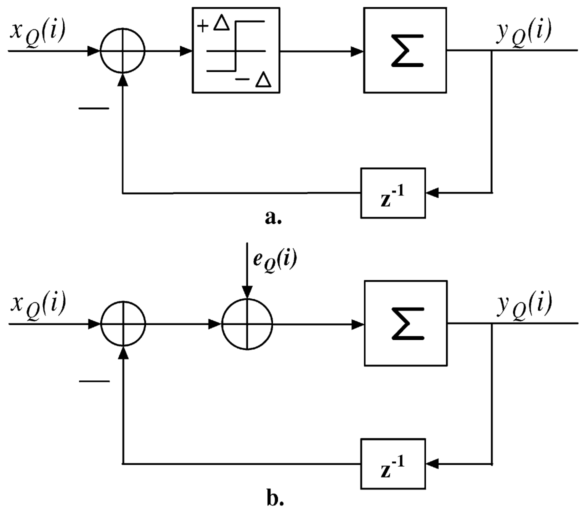

Fig. 3. Linear delta modulator: (a) single-bit structure, (b) approximation of the quantizer by an additive quantization noise.

or decrementing the output by one step at a time depending on the difference between input and output values. Using the linearized form of the DM shown in Fig. 3b, one can show that

$$Y_Q(z) = \frac{\frac{1}{1-z^{-1}}}{1+\frac{z^{-1}}{1-z^{-1}}}\left[X_Q(z)+E_Q(z)\right], \tag{9}$$

$$= X_Q(z)+E_Q(z). \tag{10}$$

In the time domain, we can write

$$y_Q(i) = x_Q(i) + e_Q(i). \tag{11}$$

For practical reasons, the number of bits assigned to y_Q must be finite. In the simulation section of this study, this quantizer is implemented using 8 bits.

2.1. Equivalent random gain for the log-quantizer

In this section, we show that the DM-based log-quantizer described by Fig. 2 can by approximated by scalar random gain with known statistics. This result will be used in the analysis of the log-LMS algorithm.

From Fig. 2 and (11), we have

$$g(i) = 2^{e_Q(i)+\log_2(|e(i)|)} = 2^{e_Q(i)}\left|e(i)\right|. \tag{12}$$

The output of the log-quantizer $y(i)$ is then given by

$$y(i) = \operatorname{sign}\left[e(i)\right]g(i) = 2^{e_Q(i)}e(i). \tag{13}$$

Denoting

$$K(i) \triangleq 2^{e_Q(i)} \tag{14}$$

gives

$$y(i) = K(i)e(i). \tag{15}$$

This expression shows that the DM-based log-quantizer can be represented by a positive scalar random gain $K(i)$ that is dependent only on the quantization noise $e_Q(i)$. Following from Fig. 1, the weight update of the DM-based log-LMS algorithm can now be written as

$$\boldsymbol{w}_i = \boldsymbol{w}_{i-1} + \mu \boldsymbol{u}_i^* K(i)e(i). \tag{16}$$

2.2. Statistics of the random gain $K(i)$

In this section, the first and second moments of the variable gain $K(i)$ are derived from (14). The quantization noise $e_Q(i)$ introduced by the DM is assumed uniformly distributed within $[-\Delta, \Delta]$. This means that the expected value of the random gain $K(i)$ is given by

$$E_K \triangleq E\{K\} = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} 2^{\eta}\, d\eta = \frac{1}{2\Delta \ln(2)}\left(2^{\Delta} - 2^{-\Delta}\right). \tag{17}$$

Similarly, the second moment of $K(i)$ can be expressed as

$$E_{K^2} \triangleq E\{K^2\} = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} 2^{2\eta}\, d\eta = \frac{1}{4\Delta \ln(2)}\left(2^{2\Delta} - 2^{-2\Delta}\right). \tag{18}$$

For example, for $\Delta = 1$ we have, $E_K = 1.082$, $E_{K^2} = 1.3525$ and the variance $\sigma_K^2 = 0.1828$. These numerical values are verified through simulation as we shall describe in Section 5.

Now let us state the following assumptions:

A1. The quantization noise $e_Q(i)$ is assumed independent of the quantizer input $e(i)$. In this case, the random gain $K(i)$ can also be assumed independent of $e(i)$.
A2. The additive noise $v(i)$ is zero-mean Gaussian noise.
A3. The input $\boldsymbol{u}_i$ is stationary with autocorrelation matrix $\mathbf{R}$.
A4. For ease of analysis, we assume in Section 3 that the proposed structure has infinite word length. Simulations, however, are carried out using finite word length for practical considerations. Simulation results show that this assumption is reasonable for suitably large number of bits.

These assumptions are helpful in the convergence analysis of the log-LMS algorithm discussed in this work as will be shown in the next sections.

Notice that the validation of assumption **A1** is not straight forward due to the nonlinearities that exist in the proposed quantizer. Although the quantizer inside the DM is one-bit, the signal $e(i)$ is not a direct input to this quantizer. This signal passes through the absolute-value and the logarithmic blocks. It is then de-correlated by the DM summation block before it gets into the quantizer, which makes this assumption reasonable. To further validate this assumption, the empirical Kolmogorov–Smirnov (K–S) algorithm was applied to test the statistical independence of the signals $e(i)$ and $e_Q(i)$. The results of this test consistently validated the assumption.

3. Error analysis of the proposed algorithm

In this section, we derive expressions for the mean and mean square of the weight errors for the proposed log-LMS algorithm. To perform the analysis, let us start from the equivalent recursion of the log-LMS algorithm given by (16). This recursion can be written as

$$\boldsymbol{w}_i = \boldsymbol{w}_{i-1} + \mu \boldsymbol{u}_i^* K(i)\left[d(i) - \boldsymbol{u}_i \boldsymbol{w}_{i-1}\right] = \boldsymbol{w}_{i-1} + \mu \boldsymbol{u}_i^* K(i)\left[\boldsymbol{u}_i \boldsymbol{w}^{\mathrm{opt}} + v(i) - \boldsymbol{u}_i \boldsymbol{w}_{i-1}\right]. \tag{19}$$

Subtracting $\boldsymbol{w}^{\mathrm{opt}}$ from both sides leads to

$$\tilde{\boldsymbol{w}}_i = \left[\mathbf{I} - \mu \boldsymbol{u}_i^* \boldsymbol{u}_i K(i)\right]\tilde{\boldsymbol{w}}_{i-1} + \mu \boldsymbol{u}_i^* K(i) v(i), \tag{20}$$

where the weight error vector $\tilde{\boldsymbol{w}}_i$ is defined as

$$\tilde{\boldsymbol{w}}_i \triangleq \boldsymbol{w}_i - \boldsymbol{w}^{\mathrm{opt}}. \tag{21}$$

3.1. Convergence in the mean

Let us take the expected value of both sides of (20) taking into account assumptions **A1**–**A3**. The result is given by

$$E_{\tilde{\boldsymbol{w}}_i} = \big[\mathbf{I} - \mu \mathbf{R} E_K\big] E_{\tilde{\boldsymbol{w}}_{i-1}}. \tag{22}$$

A sufficient and necessary condition for this recursion to go to zero asymptotically is that the coefficient matrix $[\mathbf{I} - \mu \mathbf{R} E_K]$ has all its eigenvalues within the unit circle. This can be ensured if

$$|1 - \mu \lambda_{\max} E_K| < 1, \tag{23}$$

where $\lambda_{\max}$ is the maximum eigenvalue of the autocorrelation matrix $\mathbf{R}$. This leads to the following condition on the step-size μ:

$$0 < \mu < \frac{2}{\lambda_{\max} E_K}. \tag{24}$$

Comparing this condition to that of the conventional LMS, which is $0 < \mu < 2/\lambda_{\max}$, we notice that the only difference is the appearance of the term E_K in the denominator of the upper bound of the inequality. Now we can state the following result.

Theorem 1. *The weight vector of the log-LMS algorithm given by the recursion* (4) *and using the log-quantizer of Fig.* 2 *converges asymptotically in the mean to the optimal weight vector* $\boldsymbol{w}^{\text{opt}}$ *if the step-size* μ *is chosen such that*

$$0 < \mu < \frac{2}{\lambda_{\max} E_K}. \tag{25}$$

In this case, we can say that

$$\lim_{i \to \infty} E_{\tilde{\boldsymbol{w}}_i} = \boldsymbol{w}^{\text{opt}} \tag{26}$$

and therefore the estimator is unbiased in steady-state.

3.2. Convergence in the mean-square sense

To investigate the convergence of the proposed log-LMS algorithm in the mean square sense, let us multiply both sides of (20) by their conjugate transpose as follows:

$$\begin{aligned}\tilde{\boldsymbol{w}}_i \tilde{\boldsymbol{w}}_i^* = {} & \big[\mathbf{I} - \mu \boldsymbol{u}_i^* \boldsymbol{u}_i K(i)\big] \tilde{\boldsymbol{w}}_{i-1} \tilde{\boldsymbol{w}}_{i-1}^* \big[\mathbf{I} - \mu \boldsymbol{u}_i^* \boldsymbol{u}_i K^*(i)\big] + \mu \boldsymbol{u}_i^* K(i) v(i) \tilde{\boldsymbol{w}}_{i-1}^* \big[\mathbf{I} - \mu \boldsymbol{u}_i^* \boldsymbol{u}_i K^*(i)\big] \\ & + \big[\mathbf{I} - \mu \boldsymbol{u}_i^* \boldsymbol{u}_i K(i)\big] \tilde{\boldsymbol{w}}_{i-1} \mu \boldsymbol{u}_i K(i) v^*(i) + \mu^2 K(i)^2 \big|v(i)\big|^2 \boldsymbol{u}_i^* \boldsymbol{u}_i.\end{aligned}$$

For simplicity of notation, denote

$$\mathbf{C}_i \triangleq E_{\tilde{\boldsymbol{w}}_i \tilde{\boldsymbol{w}}_i^*}. \tag{27}$$

Then, based on assumptions **A1**–**A4**, the expected values of the cross terms will disappear and therefore

$$\mathbf{C}_i = E\big\{\big[\mathbf{I} - \mu \boldsymbol{u}_i^* \boldsymbol{u}_i K(i)\big] \tilde{\boldsymbol{w}}_{i-1} \tilde{\boldsymbol{w}}_{i-1}^* \big[\mathbf{I} - \mu \boldsymbol{u}_i^* \boldsymbol{u}_i K^*(i)\big]\big\} + \mu^2 E_{K^2} \sigma_v^2 \mathbf{R}.$$

In other words,

$$\mathbf{C}_i = \mathbf{C}_{i-1} - \mu E_K (\mathbf{C}_{i-1}\mathbf{R} + \mathbf{R}\mathbf{C}_{i-1}) + \mu^2 E_{K^2} \big[\mathbf{R}\ \mathrm{Tr}(\mathbf{R}\mathbf{C}_{i-1}) + \mathbf{R}\mathbf{C}_{i-1}\mathbf{R}\big] + \mu^2 E_{K^2} \sigma_v^2 \mathbf{R},$$

where Tr( ) denotes the trace operation. In steady state, we can write

$$\mu E_K (\mathbf{C}_\infty \mathbf{R} + \mathbf{R}\mathbf{C}_\infty) - \mu^2 E_{K^2} \big[\mathbf{R}\ \mathrm{Tr}(\mathbf{R}\mathbf{C}_\infty) + \mathbf{R}\mathbf{C}_\infty \mathbf{R}\big] = \mu^2 E_{K^2} \sigma_v^2 \mathbf{R}.$$

Divide all terms by μE_K, we have

$$(\mathbf{C}_\infty \mathbf{R} + \mathbf{R}\mathbf{C}_\infty) - \alpha \big[\mathbf{R}\ \mathrm{Tr}(\mathbf{R}\mathbf{C}_\infty) + \mathbf{R}\mathbf{C}_\infty \mathbf{R}\big] = \alpha \sigma_v^2 \mathbf{R}, \tag{28}$$

where $\alpha \triangleq \mu \frac{E_{K^2}}{E_K}$. The solution for this system is given by [1]

$$\mathbf{C}_\infty = \frac{\alpha\sigma_v^2}{1-c}\big[2\mathbf{I} - \alpha\mathbf{R}\big]^{-1}, \tag{29}$$

where $c \in [0, 1)$ is given by

$$c = \sum_{j=1}^{M} \frac{\alpha\lambda_j}{2-\alpha\lambda_j}, \quad c \in [0, 1) \tag{30}$$

while $\{\lambda_j\}$ are the eigenvalues of the input covariance matrix $\mathbf{R}$. This is true if and only if

$$0 < \alpha < \frac{2}{\lambda_{\max}}. \tag{31}$$

In terms of μ, we have

$$0 < \mu < \frac{2E_K}{\lambda_{\max} E_{K^2}}. \tag{32}$$

We can also write the steady-state expression for the square error as

$$\mathrm{MSE}_{\mathrm{ss}} \triangleq \lim_{i\to\infty} E\big|e(i)\big|^2 = \frac{\sigma_v^2}{1-c} = \sigma_v^2 + \frac{c\sigma_v^2}{1-c} \tag{33}$$

and therefore, the excess MSE is then given by

$$\mathrm{MSE}_{\mathrm{ex}} \triangleq \lim_{i\to\infty} E\big|e(i)\big|^2 - J_{\mathrm{o}} = \frac{c\sigma_v^2}{1-c}, \tag{34}$$

where $J_{\mathrm{o}} = \sigma_v^2$ is the optimum performance index. The *misadjustment* can also be computed as follows:

$$\Omega = \frac{E|e(\infty)|^2 - J_{\mathrm{o}}}{J_{\mathrm{o}}}. \tag{35}$$

This expression can then be simplified as

$$\Omega = \frac{c}{1-c}. \tag{36}$$

Based on the above discussions, the following result can be stated.

Theorem 2. *Consider the log-LMS recursion* (4) *with the log-quantizer shown in Fig.* 2. *If the step-size of the recursion is chosen such that*

$$0 < \mu < \frac{2E_K}{\lambda_{\max} E_{K^2}}$$

then the recursion converges in the mean square to the quantity

$$\mathbf{C}_\infty = \frac{\alpha\sigma_v^2}{1-c}\big[2\mathbf{I} - \alpha\mathbf{R}\big]^{-1}, \tag{37}$$

where $\alpha \triangleq \mu \frac{E_{K^2}}{E_K}$ *and* $c = \sum_{j=1}^{M} \frac{\alpha\lambda_j}{2-\alpha\lambda_j}$. *Furthermore, the steady-state MSE is given by*

$$\mathrm{MSE}_{\mathrm{ss}} = \sigma_v^2 + \frac{c\sigma_v^2}{1-c} \tag{38}$$

while the expression for the excess MSE is

$$\mathrm{MSE}_{\mathrm{ex}} = \frac{c\sigma_v^2}{1-c}. \tag{39}$$

Finally, the MSE misadjustment is given by

$$\Omega = \frac{c}{1-c}. \tag{40}$$

4. Implementation of the proposed DM-based log-quantizer

In this section, we develop a simple method for implementing the proposed DM-based log-quantizer described by Figs. 2 and 3.

Using simple block diagram manipulation, the exponent block in Fig. 2 is taken inside the DM loop. In this case, it can be easily shown that the logarithmic block can be removed without changing the function of the quantizer. The resulting equivalent form is shown in Fig. 4. From this new structure, the signal $g(i)$ is now given by

$$g(i) = 2^{\sum_{k=0}^{i} s_2(k)}, \tag{41}$$

where

$$s_2(i) = \Delta \operatorname{sign}\big[\big|e(i)\big| - g(i-1)\big]. \tag{42}$$

This expression can also be written in another form as follows:

$$g(i) = g(i-1)2^{s_2(i)}. \tag{43}$$

From this expression, we can redraw the block diagram of the proposed log-quantizer as shown in Fig. 5. The only difference between this structure and that of Fig. 4 is that the accumulation block is replaced by a multiplication loop and moved after the exponent block. It is this form of the log-quantizer that will be used in the implementation as will be discussed in the following.

From (43), we notice that $g(i)$ is a power-of-two number. In binary format, this means that $g(i)$ has only one bit that is "1," while the rest are zeros. Furthermore, this bit simply shifts by Δ steps each time instance either to left or right depending on the sign of $s_2(i)$. Practically speaking, $g(i)$ should have finite number of bits according to the available word length. In this study, the word length used in simulation is 8 bits.

Therefore, the proposed log-quantizer can be implemented as shown in Fig. 6. The signal $|e(i)|$ is first compared to $g(i-1)$. If $|e(i)|$ is greater than $g(i-1)$, the signal $g(i)$ is multiplied by 2^{Δ}. This is simply done by shifting its

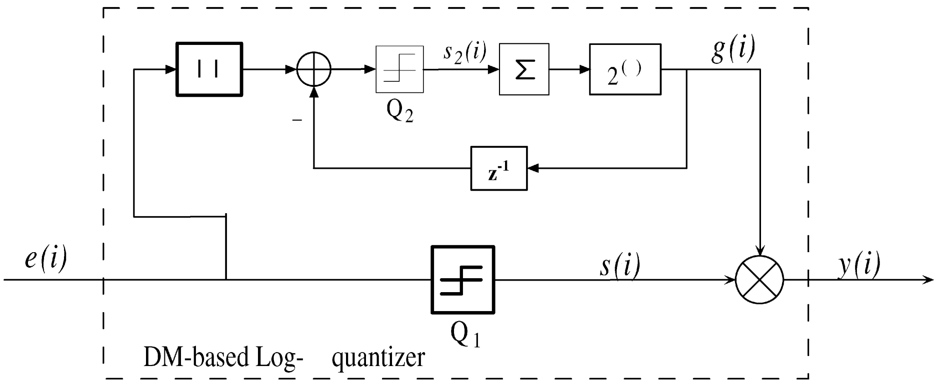

Fig. 4. Equivalent form of the DM-based log-quantizer, where the exponent block is moved inside the DM and the logarithmic block is removed.

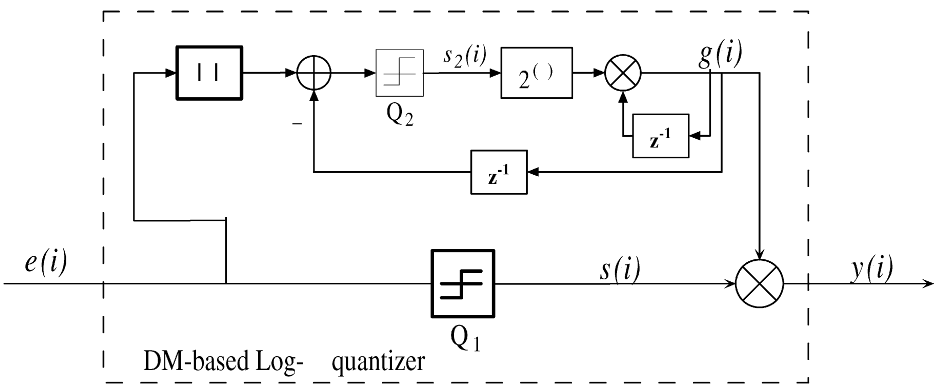

Fig. 5. Second equivalent form of the DM-based log-quantizer.

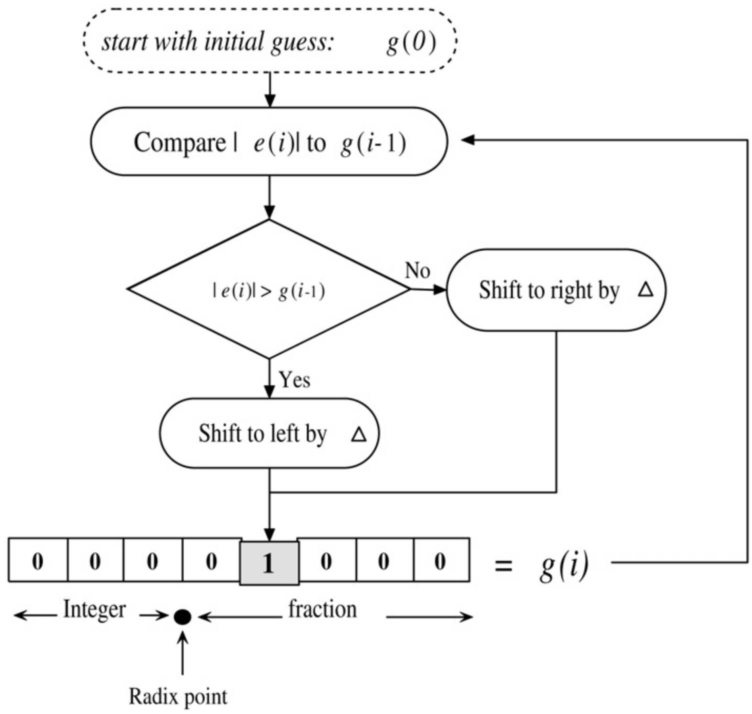

Fig. 6. Implementation algorithm for the DM-based log-quantizer.

binary digit "1" by Δ steps to the left. On the other hand, if $|e(i)|$ is less than $g(i-1)$, the signal $g(i)$ is multiplied by $2^{-\Delta}$ through shifting its binary digit "1" by Δ steps to the right. This shows that the DM-based log-quantizer can be implemented by simple shift and loop operations.

4.1. Implementation cost

As mentioned earlier, the main advantage of the logarithmic quantization concept is that it completely eliminates the need for real multiplications in the adaptive filter update equation by quantizing the error signal $e(i)$ into a power-of-two value. Basically, the proposed scheme saves half of the multiplications needed to filter and update using conventional LMS. The number of real multiplications that will be saved *per iteration* will be M and $4M$ for real and complex cases respectively. For example, if $M = 100$ taps, then the number of real multiplications saved for 10^4 iterations will be 10^6 and 4×10^6 for real and complex signaling, respectively.

5. Simulations

In this section the performance of the proposed DM-based log-LMS algorithm will be investigated via simulation. The problem considered in simulation is the identification of a 20th-order FIR system using an adaptive filter with the same order. White random regression input $u(i)$ with unit variance and zero-mean Gaussian additive noise $v(i)$ are used through out simulation. The variance of the additive noise is set to $\sigma_v^2 = 10^{-4} = -40$ dB. Finally, the learning curves of the adaptive algorithms considered in simulation are obtained by plotting the MSE versus iterations averaged over 100 runs.

First, the analytical results for the first and second moments of the variable gain $K(i)$ derived in (17) and (18) are compared with those obtained from simulation. The analytical and simulated values are listed in Table 1 for comparison, where close matching is observed.

The steady-state MSE expressions derived in (33), (34), and (36) are also compared with those obtained from simulation. The analytical and simulated steady-state MSE are shown in Fig. 7 with respect to the change in the step-size μ. Moreover, Fig. 8 shows a similar plot in terms of the noise variance. Both plots clearly show that the analytical results match those obtained via simulation.

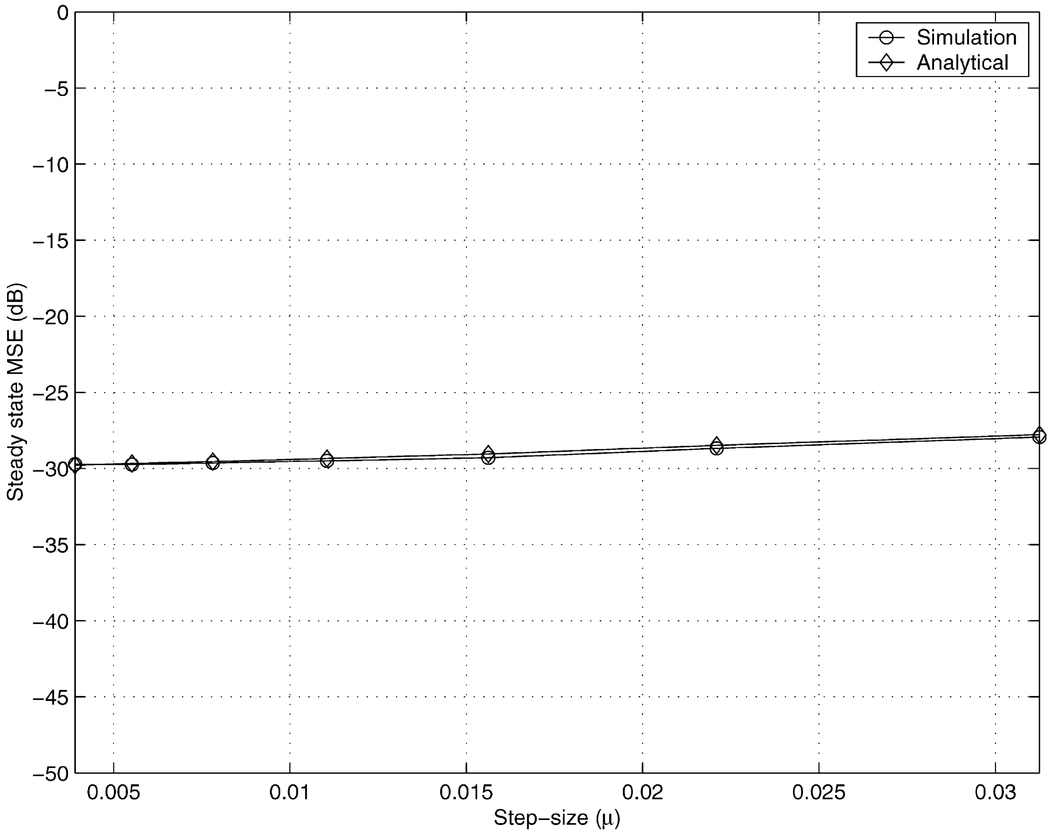

Fig. 7. The analytical and simulated steady-state MSE versus step-size with $\Delta = 1$.

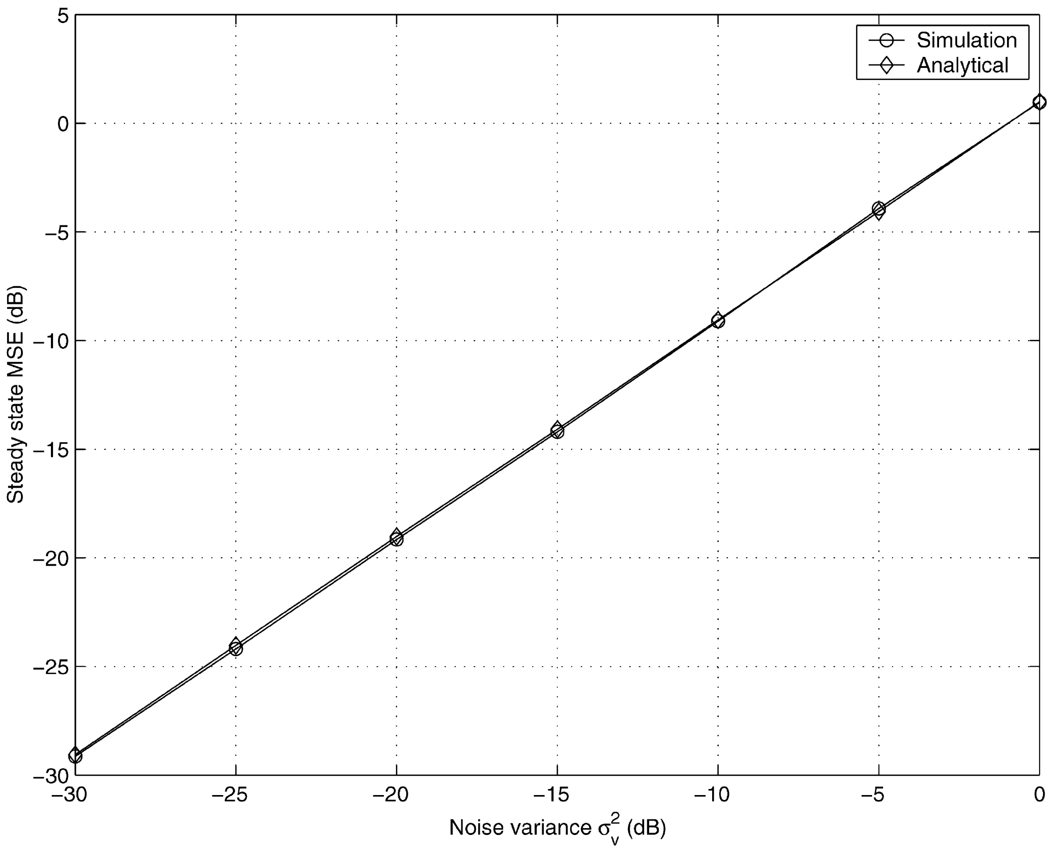

Fig. 8. The analytical and simulated steady-state MSE versus noise variance with $\Delta = 1$.

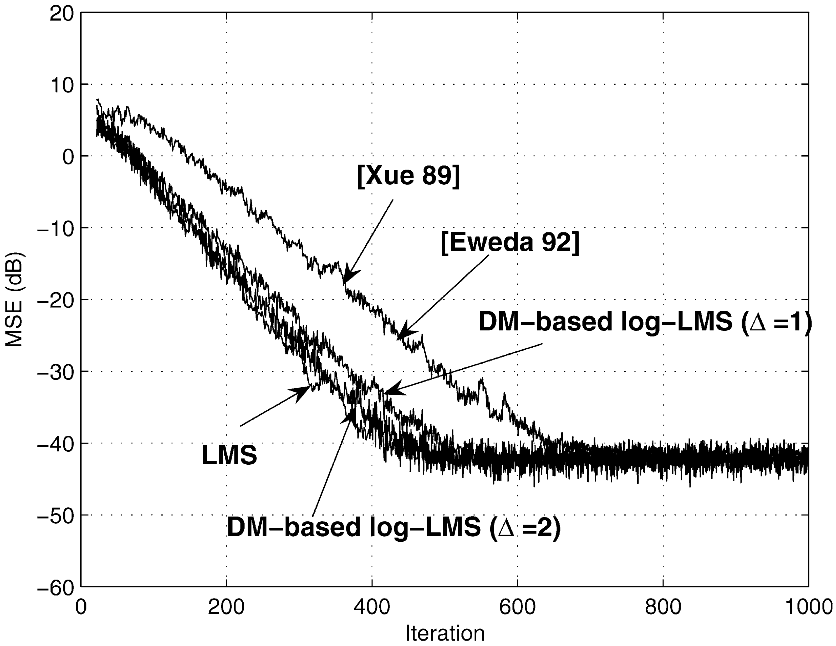

Fig. 9. Learning curves for the proposed DM-based log-LMS (with $\Delta = 1$ and 2), LMS, sign algorithm (SLMS), and the algorithms by Xue and Liu and Eweda with $B = 8$ bits and $\mu = 2^{-6}$.

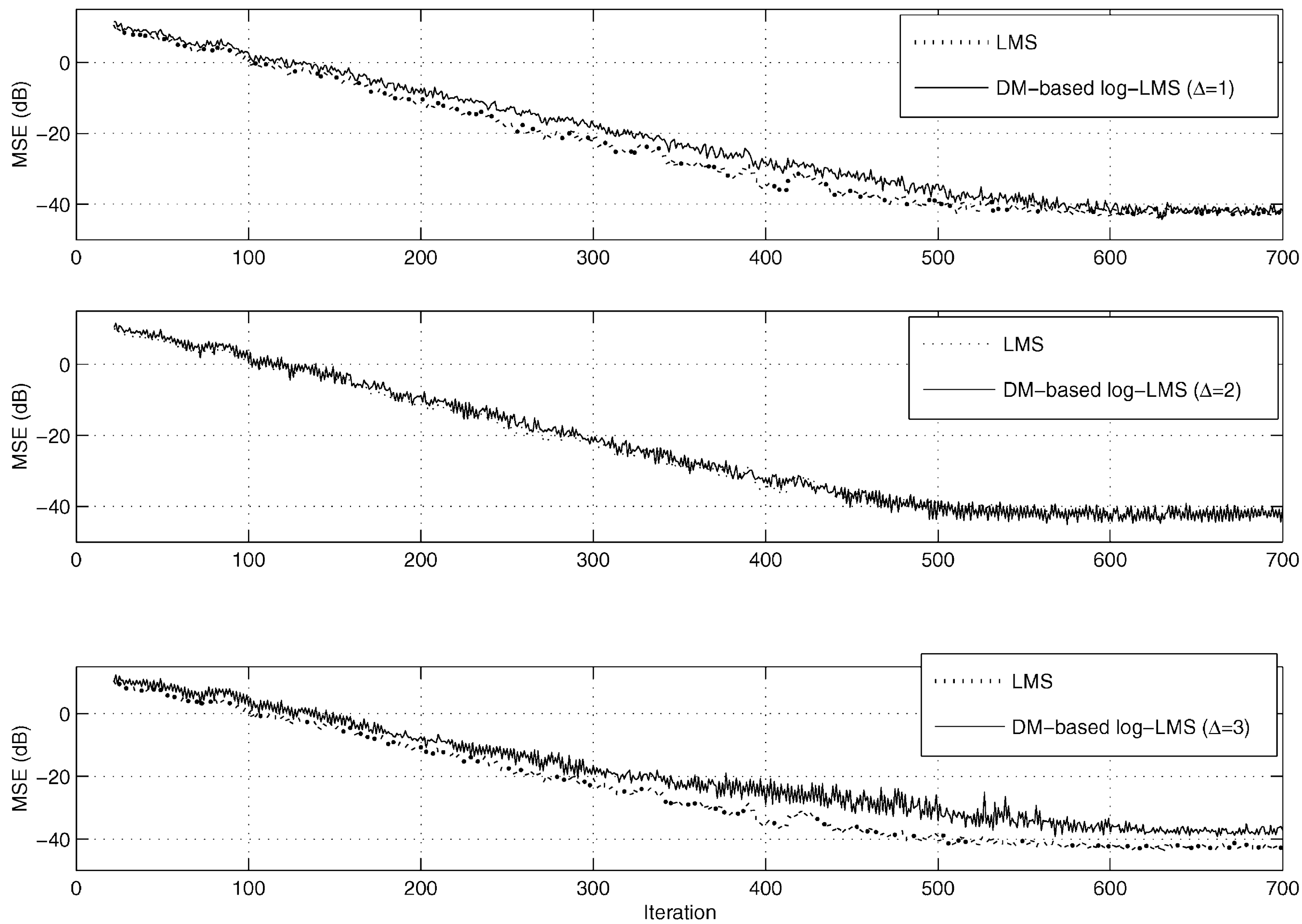

Fig. 10. Transition period of the LMS compared to that of DM-based log-LMS with different values of the step-size Δ.

Table 1
Comparison between analytical and simulated results of the first and second moments and variance of the variable gain $K(i)$ with $\Delta = 1$

	Analytical	Simulation
E_K	1.0820	1.0898
E_{K^2}	1.3525	1.3732
σ_K^2	0.2705	0.2834

The learning performance of the proposed algorithm is then compared with that of the original LMS. The two algorithms given by (6) and (7) are also included in the simulation for the purpose of comparison. The learning curves of these algorithms is computed and the results are shown in Fig. 9 with $\mu = 2^{-6}$. In this case 8-bit word-length is used for the PCM-quantizer in (6) and (7) as well as for the DM output of the proposed DM-based quantizer. The proposed algorithm clearly shows a superior performance that is much closer to that of the full precision LMS algorithm. The reason behind the fast convergence of the proposed algorithm is mainly due to the improved tracking performance of the companded DM [11]. For $\Delta = 2$, the *effective* step-size of the proposed algorithm is increased (see (16)) and hence the convergence speed is improved. In this case, the learning curve is almost identical to that of the original LMS. When Δ is increased to 3, the quantization noise increased to a level where it starts to affect the MSE performance. This is clearly indicated in the transience performance comparison shown in Fig. 10.

6. Conclusion

In this paper, a new method for logarithmic quantization LMS was proposed. This method uses a companded delta modulation structure to perform power-of two-quantization of the error signal. The convergence behavior of the proposed algorithm was investigated analytically. Closed-form expressions for the error statistics of the proposed algorithm were derived. Furthermore, an efficient method for implementing this algorithm using simple shift and loop operations was presented. Finally, simulations of the proposed algorithm supported the analytical findings and showed a performance that is comparable to that of the conventional full-precision LMS algorithm.

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