# **Number Systems Arithmetic**

## **Objectives**

➤ In this lesson, we will study basic arithmetic operations in various number systems with a particular stress on the binary system.

#### **Approach**

- ➤ Arithmetic in the Binary number system (addition, subtraction and multiplication).
- > Arithmetic in other number systems

#### **Binary Addition**

$$0 + 0 = 0$$

$$1 + 0 = 1$$

$$0 + 1 = 1$$

$$1 + 1 = 2$$

$$1 + 1 = (10)_{2}$$

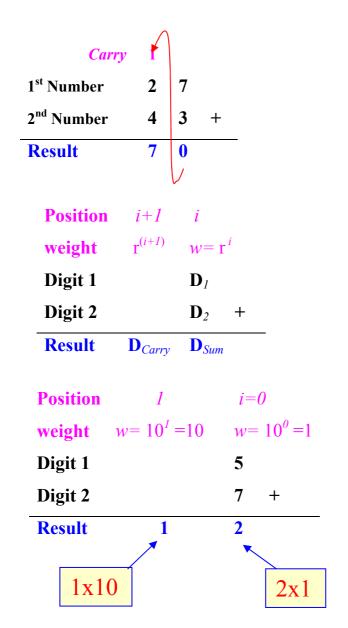
$$(3)_{10} + (7)_{10} = (ten)_{10}$$

$$(3)_{10} + (7)_{10} = (10)_{10}$$

#### **Example**

Show the result of adding:

$$(27)_{10} + (43)_{10}$$

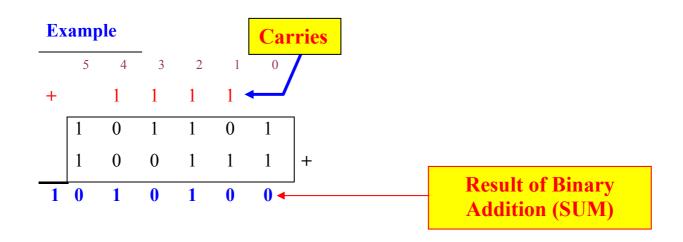


Likewise, in case of the binary system, if the weight of the sum bit is  $2^{i}$ , then the weight of the carry bit is  $2^{i+I}$ .

- Thus, adding 1 + 1 in the <u>binary</u> system results in a Sum bit of 0 and a carry bit of 1.
- ➤ The shown table summarizes the *Sum* and *Carry* results for binary addition

**Binary Addition Table** 

	•					
	Carry	Sum				
Weight	21	20				
0+0	0	0				
0 + 1	0	1				
1+0	0	1				
1+1	1	0				
$\equiv 1 \times 2^1 \qquad \equiv 0 \times 2^0$						
=1XZ = 0XZ = +2						



## **Binary Subtraction**

$$(5)_{10} - (8)_{10} = (7)_{10}$$
 Borrow 1

> For Binary subtraction

$$0 - 1 = 1$$
 Borrow 1

➤ In general, the result of subtracting two digits each of weight w is two digits. One is the "Difference" digit and the other is the "Borrow" digit.

- $\triangleright$  The **difference** digit has the <u>same weight</u> w as the operand digits.
- The **borrow** digit is considered negative and has the weight of the next *higher digit* (*wr*).

	Borrow	Difference					
Weight	<b>-2</b> <sup>1</sup>	+20					
0 - 0	0	0					
1 - 1	0	0					
1 - 0	0	1					
0 - 1	1	1					
_							
$\equiv 1x(-2^1) \qquad \equiv +1x2^0$							
	≡ -1						

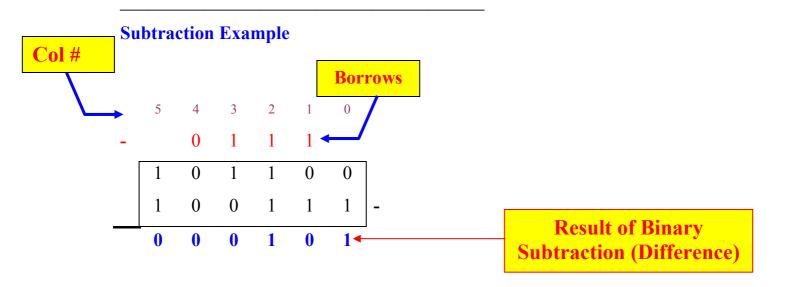
- **Q**. What is 1 1 1 = ?
- A. The answer is 1 borrow 1.

**Explanation**: We perform the operation in 2 steps:

- 1-1=0
- We then *subtract* 1 from the above result, i.e. 0 − 1 which is 1
   borrow 1.
- **Q**. What is 0 1 1 = ?
- **A**. The answer is **0 borrow 1**.

**Explanation**: We perform the operation in 2 steps:

- 0 1 = 1 borrow 1
- We then *subtract* 1 from the above result, which yields 0 borrow 1.



## **Binary Multiplication (example)**

Multiplica	ınd		1	0	1	1	
Multiplier	•			1	0	1	X
			1	0	1	1	
		0	0	0	0		+
	1	0	1	1			+
	1	1	0	1	1	1	

#### **Arith. With Bases Other Than 10**

## **Example:** Base $5 \rightarrow$ Digit Set= $\{0, 1, 2, 3, 4\}$

$$(2)_5 + (3)_5 = (5)_{10}$$
  
=  $(?)_5$   
=  $(10)_5$ 

#### **Addition Table**

+	0	1	2	3	4 =5=	$0x5^0 + 1x5^1$
0	0					
1	1	2			=6=	$1x5^0 + 1x5^1$
2	2	3	4			
3	3	4	10	11		$= 8 = 3x5^0 + 1x5^1$
4	4	10	11	12	13	0 3A3 + 1A3

## **Multiplication Table**

*	0	1	2	3	4	$=6=1x5^0+1x5^1$
0	0					
1	0	1				$=9=4x5^{0}+1x5^{1}$
2	0	2	4			
3	0	3	11	14		$=16=1x5^0+3x5^1$
4	0	4	13	22	31	