# Impact of Modeling Magnetic Saturation on the Estimation of Synchronizing and Damping Torques of Synchronous Machine

Fawzi A. Rahman AL Jowder

Electrical and Electronics Engineering Department, University of Bahrain, P.O Box 32038, Isa Town, Kingdom of Bahrain

*ABSTRACT* — **This paper investigates the influence of modeling magnetic saturation on the synchronizing and damping torques of a salient pole synchronous machine. Three models of a salient pole synchronous machine, based on state space d-q modeling, are used to perform the comparison. These models are: (1) Unsaturated state-space model, (2) Saturated state-space model ignoring the cross-saturation and (3) Saturated state-space model including cross-saturation. Using a method developed in the literature, which depends on the time responses of the rotor speed, rotor angle and electrical torque, the synchronizing and damping torques have been estimated and compared for different loading conditions using the three models of the synchronous machine. In addition, time domain simulations have been performed to examine the three models under small and large disturbances. It has been shown that including magnetic saturation in the model of synchronous machine plays an important rule in estimating the synchronizing and damping torques which in turns provide more accurate predication of the behavior of the machine under small and large disturbances.**

***Index Terms*** **— Synchronizing Torque, Damping Torque, Synchronous Machine, Magnetic Saturation.**

## I. NOMENCLATURE

| | |
|---|---|
| $v_{ds}$ | Direct-axis stator voltage |
| $v_{qs}$ | Quadrature-axis stator voltage |
| $v_f$ | Field voltage |
| $i_{ds}$ | Direct-axis stator current |
| $i_{qs}$ | Quadrature-axis stator current |
| $i_f$ | Field current |
| $i_{dm}$ | Direct-axis magnetizing current |
| $i_{qm}$ | Quadrature-axis magnetizing current |
| $m$ | Saliency factor |
| $L_{ls}$ | Leakage inductance of the stator winding |
| $L_{lf}$ | Leakage inductance of the field winding |
| $L_{lkd}$ | Leakage inductance of the d-axis damper winding |
| $L_{lkq}$ | Leakage inductance of the q-axis damper winding |
| $L_{dm}$ | Direct-axis saturated magnetizing inductance |
| $L_{qm}$ | Quadrature-axis saturated magnetizing inductance |
| $r_s$ | Stator winding resistance |
| $r_f$ | Field winding resistance |
| $r_{kd}$ | Direct-axis damper winding resistance |
| $r_{kq}$ | Quadrature-axis damper winding resistance |
| $\omega_r$ | Rotor speed |
| $\psi_u$, $i_u$ and $L_u$ | Magnetizing flux, magnetizing current and magnetizing inductance equivalent in isotropic machine. |
| $K_D$ | Damping torque coefficient |
| $K_S$ | Synchronizing torque coefficient |

## II. INTRODUCTION

Magnetic Saturation modeling has an important role in the modeling of electrical machines [1,2]. It requires information about d- and q-axis magnetizing curves. The d-axis magnetizing curve is simply the open-circuit (no-load) characteristics of the machines while the q-axis magnetizing curve is usually not available and one may assume that the level of saturation of q-axis and d-axis is the same. This approach leads to single saturation approach which converts an anisotropic synchronous machine into isotropic equivalent. This approach has been used to derive alternative models of saturated synchronous machines [3,4]. Cross-saturation is an important phenomenon which has been studied extensively in literature [5-7]. It has been shown, experimentally, the existence of this phenomenon and its impact on the steady state and transient analysis.

Synchronous machines are important element of electric drives systems and power systems and their behavior under small and large disturbances influences the whole system. Small disturbance stability is characterized by the damping and synchronizing torque coefficients [8,9]. In [10], an accurate method has been proposed to calculate these coefficients. The method is based on numerical analysis of the time responses of rotor speed, rotor angle and electrical torque using least squares error criterion.

This paper studies the influence of modeling magnetic saturation of the synchronous machine of the on the estimation of the damping and synchronizing torque coefficients. The saturated synchronous machine model developed in [3] has been used in this research. The simulations from this model and the unsaturated model of synchronous machine are used to estimate the damping and synchronizing torques using the method in [10] at different levels of active and reactive power output.

## III. SYNCHRONOUS MACHINE MODEL

In this paper, the currents in the windings are used as state variables in the d-q axis model of salient pole saturated synchronous machine in which saturation in d-axis is considered. It is assumed that the *d* and *q* axis saturates at the same degree under all operating conditions and a single saturation factor *m* is defined at all operating conditions as $m = {L_{qm}}/{L_{dm}}$ .Thus, the anisotropic salient pole machine is converted to an equivalent isotropic machine [3]. The salient pole saturated synchronous machine parameters are given in the Appendix.

### A. Saturated Synchronous machine model

The saturated synchronous machine model of a five winding can be given in a matrix form as [3]:

$$\begin{aligned}
[v] &= [A]\, {d[x]}/{dt} + [B][x] \\
[v] &= \begin{bmatrix} v_{ds} & v_f & 0 & v_{qs} & 0 \end{bmatrix} \\
[x] &= \begin{bmatrix} i_{ds} & i_f & i_{dm} & i_{qs} & i_{qm} \end{bmatrix}
\end{aligned} \qquad (1)$$

The matrices $[A]$ and $[B]$ are given as:

$$A = \begin{bmatrix} -L_{ls} & 0 & -L_{dd} & 0 & -L_{dq} \\ 0 & L_{lf} & L_{dd} & 0 & L_{dq} \\ -L_{lkd} & -L_{lkd} & (L_{lkd} + L_{dd}) & 0 & L_{dq} \\ 0 & 0 & -L_{dq} & -L_{ls} & L_{qq} \\ 0 & 0 & L_{dq} & -L_{lkq} & (L_{lkq} + L_{qq}) \end{bmatrix} \qquad (2)$$

$$B = \begin{bmatrix} -r_s & 0 & 0 & -\omega_r L_{ls} & -\omega_r L_{qm} \\ 0 & r_f & 0 & 0 & 0 \\ -r_{kd} & -r_{kd} & r_{kd} & 0 & 0 \\ \omega_r L_{ls} & 0 & \omega_r L_{dm} & -r_s & 0 \\ 0 & 0 & 0 & -r_{kq} & r_{kq} \end{bmatrix} \qquad (3)$$

The saturation dependent inductances of equations (2) and (3) are given as:

$$\begin{aligned}
L_{dd} &= L\cos^2 u + L_u \sin^2 u \\
Lqq &= m(L\sin^2 u + L_u \cos^2 u) \\
L_{dq} &= m(L - L_u)\sin u \cos u
\end{aligned}$$

where

$$\begin{aligned}
L &= L_{dm} = \frac{\psi_u}{i_u} \\
L_u &= \frac{d\psi_u}{di_u} \\
\sin\mu &= {mi_{qm}}/{i_m} \\
\cos\mu &= {i_{dm}}/{i_m} \\
i_m &= \sqrt{(i_{md}^2 + i_{mq}^2)}
\end{aligned} \qquad (4)$$

### B. Omission of Cross Saturation Representation in Saturated Synchronous Machine Model

The cross-saturation is neglected by setting the term $L_{dq}$ in (2) to zero. This is achieved by making $L$=$L_u$ in (2). Thus, the saturation dependent inductances are given as:

$$\begin{aligned}
L_{dd} &= L_u \\
L_{qq} &= m^2 L_u \\
L_{dq} &= 0
\end{aligned} \qquad (5)$$

The synchronous machine model, neglecting the cross saturation, can be obtained by substituting (5) in (2).

## IV. METHODS OF CALCULATING SYNCHRONIZING AND DAMPING POWER COEFFICIENTS

In [10], a method of calculating the synchronizing and damping torque coefficient of a synchronous machine is developed. The method is based on the numerical analysis of the system responses using least square method. The needed system responses are rotor speed, rotor angle and electrical torque responses to an impulse response in the electrical torque. The method is briefly explained to make the paper self-sufficient.

The incremental change in electrical torque is given as:

$$\Delta T_e(t) = K_D \Delta\omega(t) + K_S \Delta\delta(t) \qquad (6)$$

where,

$K_D$ is the damping torque coefficient
$K_S$ is the synchronizing torque coefficient

If the time responses of the electrical torque, rotor angle and rotor speed are available, then the sum of error E(t) is given as:

$$\sum E(t) = \sum\left(\Delta T_e(t) - [K_D \Delta\omega(t) + K_S \Delta\delta(t)]\right) \qquad (7)$$

The damping and synchronizing torque coefficients $K_D$ and $K_S$ are calculated using least square method, i.e. minimizing the square of the sum of error given in (2). The coefficients should satisfy $\partial E^2 / \partial K_D = 0$ and $\partial E^2 / \partial K_S = 0$ . The following expressions are obtained:

$$\begin{aligned}\sum \Delta T_e(t) \times \Delta\delta(t) &= K_S \sum \Delta\delta^2(t) + K_D \sum (\Delta\delta(t) \times \Delta\omega(t)) \\ \sum \Delta T_e(t) \times \Delta\omega(t) &= K_S \sum (\Delta\delta(t) \times \Delta\omega(t)) + K_D \sum \Delta\omega^2(t)\end{aligned} \quad (8)$$

Having the time responses of $\Delta\omega$, $\Delta\delta$ and $\Delta T_e$, the coefficients $K_D$ and $K_S$ are calculated.

A five winding salient-pole synchronous generator, whose data are given in the Appendix, is simulated to estimate the synchronizing and damping torques coefficients of different models. Fig. 1 shows, respectively, the time responses of the incremental change in the rotor speed $\Delta\omega$ , the incremental change in rotor angle $\Delta\delta$ and the incremental change in electrical torque $\Delta T_e$ due to impulse response in the reference mechanical power. Fig. 1(c) shows that estimated incremental change in electrical torque using the explained method (dotted line) matches with the measured incremental change in electrical torque $\Delta T_e$ (sold line).

Fig.2, Fig. 3 and Fig. 4 show, respectively, the damping and synchronizing torque coefficients as functions of the reactive power for different levels of active power. Negative reactive power indicates leading power factor while positive power factor indicates lagging power factor. It can be seen from the figures that the damping torque coefficient decreases as the reactive power increases. On the other hand, the synchronizing torque coefficient increases as the reactive power increases. This is because the excitation is the main contributor to the synchronizing torque [11].

Fig. 5 compares between the damping and synchronizing torque coefficients using unsaturated model, saturated model ignoring cross-saturation and saturated model including cross-saturation. It can be depicted from Fig. 5a that including saturation in the synchronous machine model increases the damping torque coefficient at the leading power factor while it decreases the damping torque coefficient at lagging power coefficient. It can be also seen from Fig 5a that ignoring cross-saturation gives less damping torque coefficient at leading and lagging power factors that the case when it is included. Fig5b shows that the synchronizing torque coefficients of the saturated models are larger than the unsaturated values at both leading and lagging. This is because of the reduction in the mutual inductance which increases the synchronizing torque coefficient. Higher synchronizing torque coefficients are obtained when cross-saturation is included in the saturated model.

## V. Simulation Results

Two tests have been performed to verify the results obtained in the previous section. The first test is a 5% step change in the terminal voltage, which is given in Fig. 6, while the second test is a three-phase fault at the machine terminal, which is given in Fig.7. Fig. 6 (a) and (b) show, respectively, the rotor speed and angle responses at heavy loading conditions ($P_{out}$=1.0 pu, $Q_{out}$=1.0 pu). It can be seen from Fig. 6 that the unsaturated model provides more damping than the saturated models. Fig. 6(b) shows that the saturated model with cross saturation provides higher synchronizing torque since the synchronizing torque is inversely proportional to the rotor angle ($K_s = \frac{\partial P_{ac}}{\partial \delta} = \frac{V^2}{X}\cos\delta$) . Fig. 7 (a) and (b) show, respectively, the rotor speed and angle responses due to large disturbance (three-phase fault). It can be observed from Fig. 7 that the machine is unstable if the saturated model ignoring cross saturation is used in the simulation due to lack of damping. However, it is stable when the other two models are used in the simulation.

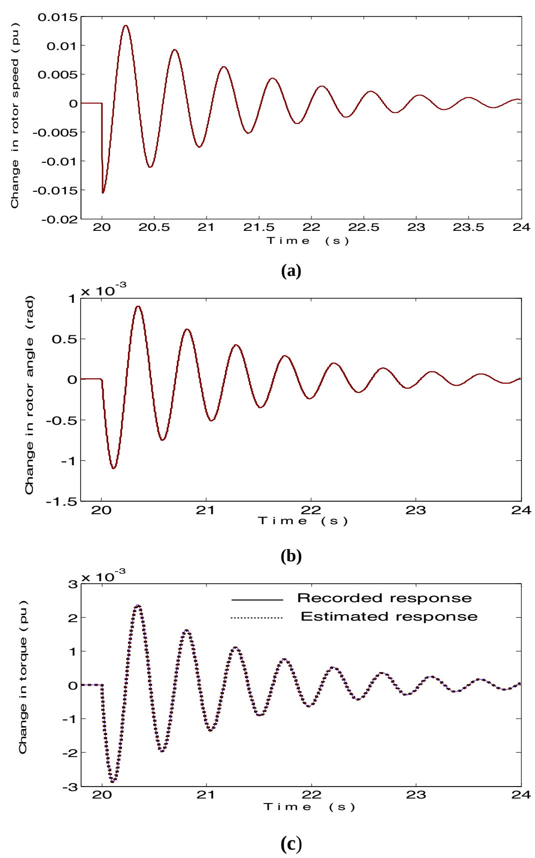
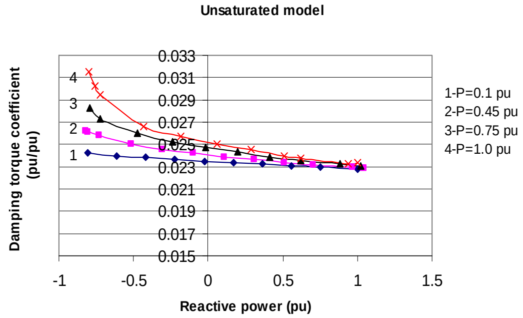



**Fig.1 Time responses of rotor speed, rotor angle and electrical torque**

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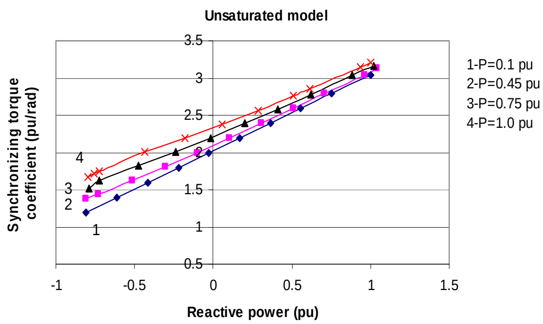
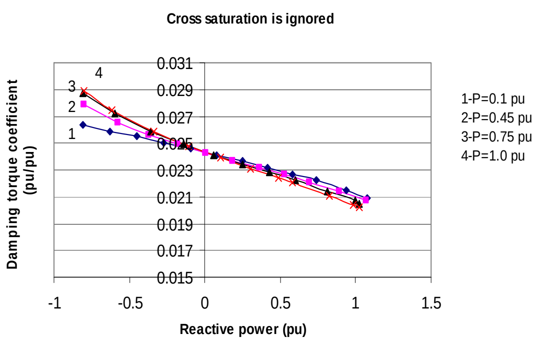



**(b)**

**Fig. 2 Damping and synchronizing torques coefficients using unsaturated model.**

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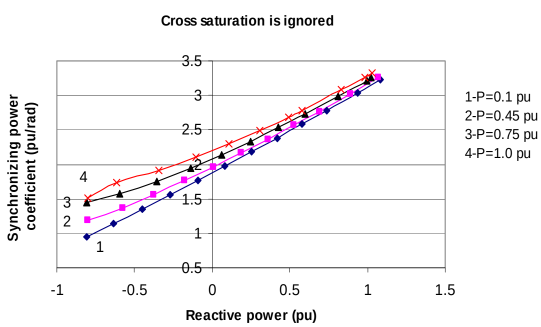
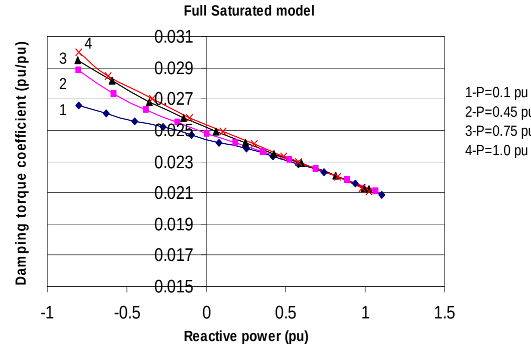



**(b)**

**Fig. 3 Damping and synchronizing torques coefficients using saturated model ignoring cross-saturation.**

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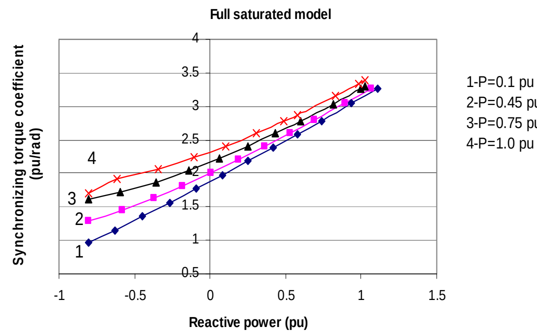
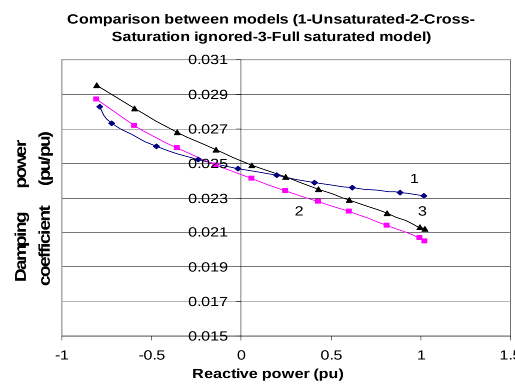



**(b)**

**Fig.4 Damping and synchronizing torques coefficients using saturated model.**

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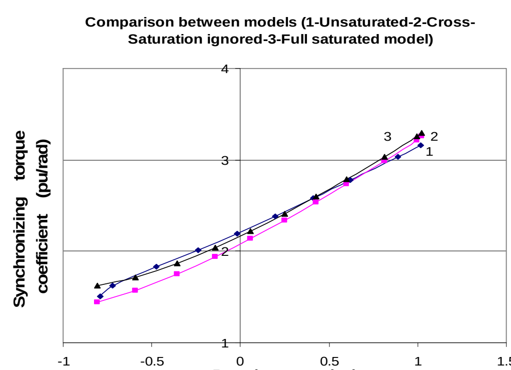
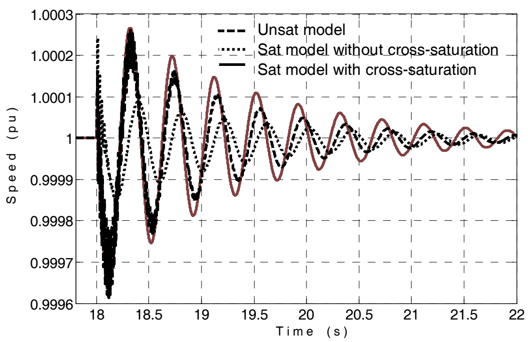



**(b)**

**Fig. 5 Comparison between the three models.**

## VI. Conclusion

In this paper, it has been found that including saturation in the synchronous machine model increases the damping torque coefficient at the leading power factor while it decreases the damping torque coefficient at lagging power coefficient. It has been also found that synchronizing torques coefficients of the saturated models are larger than the unsaturated values at both leading and lagging. Ignoring cross-saturation in the saturated model provides less synchronizing and damping torques than the case when the cross-saturation is included in the model. Thus, in order to have more accurate calculation of synchronizing and damping torques of synchronous machines, the magnetic saturation, including cross saturation, has to be modeled.

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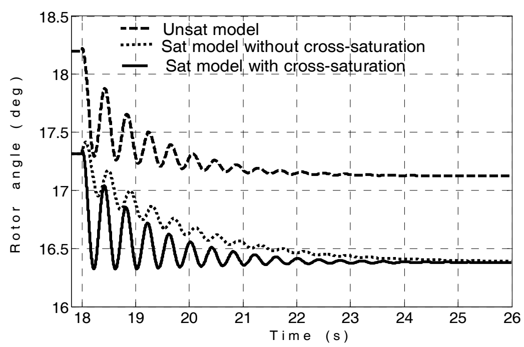
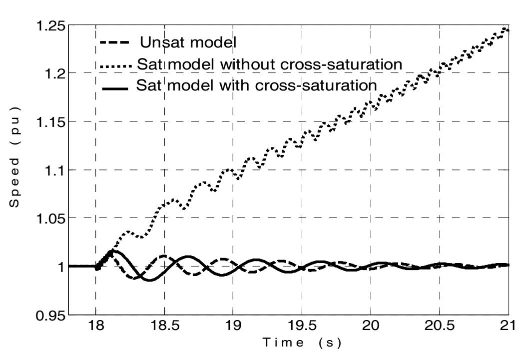



**(b)**

**Fig. 6 Synchronous machine response due to 5% step change in terminal voltage. ($P_{out}$=1.0 pu, $Q_{out}$=1.0 pu)**

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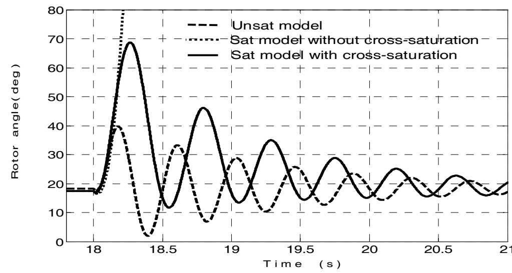


**(b)**

**Fig. 7 Synchronous machine response due to three-phase fault at the machine terminals. ($P_{out}$=1.0 pu, $Q_{out}$=1.0 pu).**

## VII. Appendix

$$R_s = 0.0062, R_f = 0.000752, R_{kd} = 0.0254, R_{kq} = 0.055, L_{ls} = 0.14,$$
$$L_{lf} = 0.2244, L_{lkd} = 0.3426, L_{lkq} = 0.242, L_{dmo} = 0.91, L_{qmo} = 0.53,$$
$$m^2 = 0.58242, H = 2.7093, f = 60Hz, \psi_u = 1.1212 \times 0.8029^{i_u} i_u^{1.0826}$$

## IX. References


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