On the Capacity-Fairness Tradeoff in Multiuser Diversity Systems*

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Abstract—**Conventional multiuser diversity maximizes the system throughput but results into an unfair scheduling of the system resources across users. Proportional fair scheduling achieves strict fairness among different users but this fairness comes at the cost of a significant system capacity penalty. In this work, we propose two forms of a hybrid multiuser scheduling scheme that provides a flexible balance/tradeoff between the system achievable capacity and the fairness among users. Our results show that the capacity vs. fairness tradeoff can be achieved by grouping users then using a two step selection process.**

Index Terms—**(1) Fading Channels, (2) Multiuser Selection Diversity, (3) Capacity, (4) Degree of Fairness, (5) Scheduling.**

I. Introduction

The third generation (3G) cellular networks are currently being deployed worldwide. While this latest generation continues to improve the quality of voice communications, a thrust of design and research efforts is to make high data rate applications really take off. Unlike voice traffic, packet data traffic can often tolerate relatively larger latency, which provides designers with additional flexibility to achieve a higher data throughput by exploiting for example multiuser diversity in a fading environment.

In a wireless multiuser communication scenario, the channel between the base station and each user experiences independent variations due to fading. This can be viewed as a form of multiuser diversity in that it is unlikely for all users to be in deep fade and therefore the communication can often occur over a strong channel. Recent studies on this multiuser diversity were motivated by [1], which shows that the total uplink (mobile to base) capacity can be maximized by picking the user with the best channel to transmit, which is often referred to as the multiuser selection diversity scheme. The study of [1] was extended to the downlink in [2] which showed that the same access scheme is valid also for the downlink case.

Allotting all resources to the user with the best channel condition at a given time slot achieves the maximum system throughput, but the fairness issue arises in that the users close to the base station will monopolize the system resource. To take advantage of multiuser diversity while still maintaining fairness among all users, a proportional fair scheduling scheme was proposed in [3]. Rather than choosing the user with the "absolute" best channel, the proportional fair scheduler selects the user with the best "normalized" channel (i.e. the channel that is experiencing the best fading condition compared to its own average). A variant of this scheduling algorithm was also proposed in [4] by taking into consideration the trade-off between the multiuser diversity gain and the mobility of users. The concept of multiuser diversity has also been included in a major 3G standard namely the CDMA2000-EVDO standard [5] in which the specific implementation of the scheduling algorithms is left to service providers to determine.

The loss in throughput by proportional fair scheduling can be significant in a typical scenario in which users are scattered across the cell [6]. Indeed, throughput-maximization and proportional fair scheduling can be viewed as two extreme scheduling procedures. The former maximizes the system throughout without any fairness consideration,while the latter achieves strict fairness among all users but at the expense of a significant loss in capacity. It is therefore natural to look for schemes that can bridge the gap between these two extremes, which is essentially the main motivation behind this work.

In this work, we first propose two forms of a *hybrid multiuser scheduling* scheme in which the user-selection is done in two steps. First, all users are divided into a number of groups. One user per group is selected based on the best "channel strength" (or "relative channel strength") criterion and then a user with the best "relative channel strength" (or "channel strength") among the chosen users is selected as the one to communicate with the base station. We then show by analytical and numerical results that these proposed schemes (i) include both the maximum throughput scheduling and the proportional fair scheduling as two extreme schemes, and (ii) can achieve a flexible balance/tradeoff between fairness and multiuser diversity gain by adjusting the number of groups.

The remainder of this paper is organized as follows. Section II describes the system model. Section III introduces the two forms of our newly proposed hybrid multiuser scheduling scheme. Section IV studies the performance of the capacity versus fairness of these proposed schemes. Some numerical and simulation results are presented and discussed in section V. Finally, section VI ends this paper with some concluding remarks.

II. System Model and Problem Statement

A. System and Channel Models

We consider a multiuser diversity system in a single-cell where L users are communicating with a base station. The downlink (base station to mobiles) channel model can be written as

$$r_i(t) = h_i(t)x(t) + n_i(t), \qquad i = 1, 2, \cdots, L, \tag{1}$$

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where $x(t) \in \mathcal{C}$ is the transmitted signal in time slot t and $r_i(t) \in \mathcal{C}$ is the received signal of user i in time slot t. It is assumed that $x(t)$ has the average (normalized) transmit power $E\left(|x(t)|^2\right) = 1$. $\{n_i(t)\}$ is an independent and identically distributed (i.i.d.) sequence of zero mean complex Gaussian noise with variance σ_n^2. $h_i(t)$ is the fading channel gain from the base station to the ith user in time slot t. We adopt the quasi-static fading channel model where $h_i(t)$ is i.i.d. from burst to burst but remains constant over each burst. We consider the flat Rayleigh fading model, assume that the fading coefficients of all users are independent but allow these coefficients not to be necessarily identically distributed. Therefore $h_i(t)$ is a zero mean complex Gaussian random variable. The amplitude of $h_i(t)$

$$\alpha_i(t) = \sqrt{|h_i(t)|^2} \tag{2}$$

is Rayleigh distributed with the probability density function (PDF) given by

$$f_{\alpha_i}(\alpha) = \frac{2\alpha}{\Omega_i} \exp\left(-\frac{\alpha^2}{\Omega_i}\right), \ \alpha \geq 0, \tag{3}$$

where Ω_i is the short-term average fading power of the ith user.

B. Sum Rate and Multiuser Diversity Scheduling

In this section, we review some information-theoretic concepts given in [1] and [2]. If both transmitter and receiver can track the channel perfectly, it is well known that the sum rate conditional on channel realization $\mathbf{h} = [h_1(t), \cdots, h_L(t)]$ is given by

$$\sum_{i=1}^{L} R_i < \log_2\left(1 + \frac{\sum_{i=1}^{L} P_i |h_i(t)|^2}{\sigma_n^2}\right) \tag{4}$$

under the normalized average transmit power constraint

$$E_{\mathbf{h}}\left(\sum_{i=1}^{L} P_i\right) = 1. \tag{5}$$

The sum rate in (4) can be maximized by transmitting to the user with the largest $|h_i(t)|^2$ at any given time slot t, i.e.,

$$\sum_{i=1}^{L} R_i < \log_2\left(1 + \frac{\mu(\mathbf{h}) \max_i |h_i(t)|}{\sigma_n^2}\right). \tag{6}$$

The optimum power allocation $\mu(\mathbf{h})$ is given by the waterfilling solution over channel states. In the rest of paper, however, we assume that, instead of using the optimum water-filling power allocation, the base station transmits with constant power 1, which is often the case for a practical cellular network. Furthermore, this constant power assumption makes some analysis easier without fundamentally changing relationship between the system throughput and the fairness that is the central issue considered in this paper.

Although scheduling the channel access to the best user maximizes the system throughput, it can be highly "unfair" when users in the system have very disparate channel conditions, i.e., users that are close to the base station in average have better channel and will end up monopolizing the channel access most of time. As a remedy, the proportional fair scheduling scheme proposed in [3] transmits to the user with the best channel condition relative to its own average, i.e., the user k^* with the largest $\frac{R_k(t)}{T_k(t)}$, where $R_k(t)$ and $T_k(t)$ are the requested data rate and the average throughput of users k, respectively. Alternatively, the base station transmits to user k^* with the largest $\frac{\gamma_k(t)}{\bar{\gamma}_k}$, where $\gamma_k(t) \triangleq \frac{\alpha_k^2(t)}{\sigma_n^2}$ is the instantaneous (in time slot t) received SNR and $\bar{\gamma}_k \triangleq \frac{\Omega_k}{\sigma_n^2}$ is the short-term average received SNR for the user k, respectively. This scheduler uses the same idea as the original proportional fair scheduling algorithm in [3] except that it operates based on the SNR criterion rather than the data rate.

C. A Fairness Measure

A definition that can quantify how fair the system resource is allocated among all users seems difficult. We adopt the fairness notion developed in [7] to compare the fairness of different multiuser diversity scheduling algorithms.

We assume that all users are equally important and have the same quality of service requirements. Then the self-fairness of a given user i is defined as [7]

$$\mathfrak{f}_i = \frac{-\log(P_i)}{-\log(1/L)} = \frac{-\log(P_i)}{\log(L)}, \tag{7}$$

where P_i is the proportion of resources (e.g., the amount of the time slots for transmission in the multiuser system) allocated to user i, or equivalently the access probability for the multiuser system, and $\log(L)$ term is a normalization factor. The average fairness of a system with L users is then defined as

$$\bar{\mathfrak{f}} = \sum_{i=1}^{L} P_i \mathfrak{f}_i = -\sum_{i=1}^{L} P_i \frac{\log(P_i)}{\log(L)}. \tag{8}$$

Thus to evaluate the self-fairness and the average fairness, we need to compute the access probability of the individual users. In this fairness definition, a system is strictly fair if every user has equal probability $\frac{1}{L}$ to access the channel regardless their average channel condition and in such case the system degree of fairness by the above definition can be calculated as 1. On the other hand, if the channel usage is dominated by a single user, then the system degree of fairness can be calculated to be 0.

It can be easily shown that if the scheduling is based on the SNR, the optimum scheduling and the proportional fair scheduling will be equivalent if all users are subject to i.i.d. fading process. However, if users have disparate channels, the throughput and fairness gaps between the optimum scheduling and the proportional fair scheduling become large. These two scheduling schemes can be viewed as two extremes: The optimum scheduling maximizes the system throughput without considering the fairness while the proportional fair scheduling maintains strict fairness among all users at the cost of throughput loss. It is therefore interesting to look at possible ways of achieving flexible tradeoff points between these two performance measures. One way to achieve this is to use the hybrid scheduling which will be described in detail in what follows.

III. Hybrid Multiuser Diversity Scheduling

The hybrid multiuser diversity scheduler first divides all active users in the system into K groups and then

1) selects one user per group based on either the "best channel" (or "relative channel strength") criterion.
2) chooses one user from all K selected users based on either the "relative channel strength" (or "best channel") criterion. This will be the user to communicate with the base station.

In this work, we refer to the conventional multiuser selection diversity as "absolute scheduling" and the proportional fair scheduling as "normalized scheduling". The hybrid multiuser scheduling scheme can have two possible combinations. One is to first employ the absolute scheduling followed by the normalized scheduling as a second step, which we term "A/N hybrid scheduling". The second combination consists of first applying normalized scheduling and then using absolute scheduling, this is referred to as the "N/A hybrid scheduling". It is easy to see that if the system takes the same scheduling scheme in both steps, the proposed hybrid scheduling scheme is equivalent to the traditional absolute/normalized scheduling scheme. It should be also noted that when the number of groups equals to one, the A/N and N/A hybrid scheduling reduce to the traditional absolute scheduling and normalized scheduling schemes, respectively. When the number of groups equals to the number of total users in the system (i.e. there is exactly one user in each group), the A/N and N/A hybrid scheduling schemes reduce to the traditional normalized scheduling and absolute scheduling, respectively. In the following, we will study the capacity versus fairness performance of the N/A and N/A hybrid scheduling schemes.

IV. Capacity vs. Fairness Performance Analysis

We consider a multiuser diversity system in a single-cell where the total number of L users are subdivided in K groups communicating with a base station. Each individual user is subject to independent but not necessarily identically distributed Rayleigh fading. Extension to the Nakagami fading scenario is straightforward. As argued in [4], a base station is usually limited by the peak power rather than the long term average power which is typical in battery-limited applications. Therefore, we assume that the transmitting power is constant over all time slots.

Let N_i denote the number of users in the ith group, $\gamma_{i,j}$ and $\bar{\gamma}_{i,j}$ be the instantaneous received SNR and short-term average received SNR for the jth user in the ith group, respectively. Then we have $\sum_{i=1}^{K} N_i = L$. For the N/A hybrid scheduling, the scheduler first selects the user j^* in ith group (where $i = 1, \cdots, K$) with largest $\frac{\gamma_{i,j}}{\bar{\gamma}_{i,j}}$, then picks the user with the largest instantaneous received SNR among the users chosen in the first step. While, for the A/N hybrid scheduling, the scheduler reverses the selection process and first selects the user j^* in the ith group (where $i = 1, \cdots, K$) with largest $\gamma_{i,j}$, then picks the user with the largest $\frac{\gamma_k}{\bar{\gamma}_k}$ (where γ_k and $\bar{\gamma}_k$ are the instantaneous received SNR and the short-term average received SNR for the user picked in the kth group in the first step, respectively) among the users chosen in the first step.

A. System Capacity

The system capacity achieved by the proposed hybrid scheduling is given by

$$C_1 = \int_0^\infty \log_2(1+\gamma_s) f_{\gamma_s}(\gamma_s) d\gamma_s, \tag{9}$$

where γ_s denotes the system output SNR, $f_{\gamma_s}(\cdot)$ is the probability density function (PDF) of γ_s. To evaluate the system capacity of the proposed scheduling schemes, we need the PDF of the system output SNR.

1) N/A Hybrid Scheduling: In Step 1, the scheduler picks the user with largest $\frac{\gamma_{i,j}}{\bar{\gamma}_{i,j}}$ in the ith group. Again let γ_i and $\bar{\gamma}_i$ be the instantaneous received SNR and the short-term average received SNR of this user. The PDF of γ_i can be shown to be given by

$$\begin{aligned} f_{\gamma_i}(\gamma) &= \sum_{j=1}^{N_i} \frac{1}{\bar{\gamma}_{i,j}} f_{i,j}\left(\frac{\gamma}{\bar{\gamma}_{i,j}}\right) \prod_{\substack{k=1\\k\neq j}}^{N_i} F_{i,k}\left(\frac{\gamma}{\bar{\gamma}_{i,j}}\right) \\ &= \sum_{j=1}^{N_i} \frac{1}{\bar{\gamma}_{i,j}} \sum_{k=0}^{N_i-1} (-1)^k \binom{N_i-1}{k} \exp\left[-(1+k)\frac{\gamma}{\bar{\gamma}_{i,j}}\right], \end{aligned} \tag{10}$$

where in the last equality we gave out the specific expression for the Rayleigh fading case. In (10), $f_{i,j}(\cdot)$ and $F_{i,j}(\cdot)$ are the PDF and cumulative distribution function (CDF) of the normalized SNR $\frac{\gamma_{i,j}}{\bar{\gamma}_{i,j}}$ of each individual user. In the second step, by employing the absolute scheduling, the scheduler further selects the final user to communicate within the K selected user in first step. Therefore, the PDF of the system output SNR γ_s, which is the instantaneous SNR of the final user, can be shown to be given by

$$f_{\gamma_s}(\gamma) = \sum_{i=1}^{K} f_{\gamma_i}(\gamma) \prod_{\substack{n=1\\n\neq i}}^{K} F_{\gamma_n}(\gamma), \tag{11}$$

where $F_{\gamma_n}(\cdot)$ is the CDF of γ_n and can be obtained simply by taking integral of the corresponding PDF. Finally, plugging the PDF and CDF into Eqn. (11), one can get the PDF of the system output SNR as

$$\begin{aligned} f_{\gamma_s}(\gamma) = \sum_{i=1}^{K} &\left\{ \sum_{j=1}^{N_i} \frac{1}{\bar{\gamma}_{i,j}} \sum_{k=0}^{N_i-1} (-1)^k \binom{N_i-1}{k} \exp\left[-(1+k)\frac{\gamma}{\bar{\gamma}_{i,j}}\right] \right\} \\ &\prod_{\substack{n=1\\n\neq i}}^{K} \left\{ \sum_{l=1}^{N_n} \sum_{m=0}^{N_n-1} (-1)^m \binom{N_n-1}{m} \frac{1}{1+m} \right. \\ &\left. \left[1-\exp\left[-(1+m)\frac{\gamma}{\bar{\gamma}_{n,l}}\right]\right]\right\}. \end{aligned} \tag{12}$$

Substituting (12) into (9), one can get the capacity of the multiuser system employing the N/A hybrid scheduling.

2) A/N Hybrid Scheduling: In Step 1, the scheduler picks the user with largest $\gamma_{i,j}$ in the ith group. The PDF of γ_i of the picked user in the ith group can be shown to be given by

$$\begin{aligned} f_{\gamma_i}(\gamma) &= \sum_{j=1}^{N_i} f_{\gamma_{i,j}}(\gamma) \prod_{\substack{k=1\\k\neq j}}^{N_i} F_{\gamma_{i,k}}(\gamma) \\ &= \sum_{j=1}^{N_i} \frac{1}{\bar{\gamma}_{i,j}} \exp\left(-\frac{\gamma}{\bar{\gamma}_{i,j}}\right) \prod_{\substack{k=1\\k\neq j}}^{N_i} \left[1-\exp\left(-\frac{\gamma}{\bar{\gamma}_{i,k}}\right)\right], \end{aligned} \tag{13}$$

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where in the last equality we gave out the specific expression for the Rayleigh fading case. In (13), $f_{\gamma_{i,j}}(\cdot)$ and $F_{\gamma_{i,j}}(\cdot)$ are the PDF and CDF of the received SNR $\gamma_{i,j}$ of each individual user. In the second step, by employing the normalized scheduling, the scheduler further selects the final user to communicate within the K selected user in first step. Therefore, the PDF of the system output SNR γ_s can be shown to be given by

$$f_{\gamma_s}(\gamma) = \sum_{i=1}^{K} \frac{1}{\bar{\gamma}_i} f_i(\frac{\gamma}{\bar{\gamma}_i}) \prod_{\substack{l=1 \\ l\neq i}}^{K} F_l\left(\frac{\gamma}{\bar{\gamma}_i}\right), \tag{14}$$

where $f_i(\cdot)$ and $F_i(\cdot)$ are the PDF and CDF of the normalized SNR $\frac{\gamma_i}{\bar{\gamma}_i}$ of the user picked in the ith group, and $\bar{\gamma}_i$ is the average SNR of the user picked in the ith group which can be obtained by $\bar{\gamma}_i = \int_0^\infty \gamma f_{\gamma_i}(\gamma) d\gamma$. The PDF $f_i(\cdot)$ can be obtained by applying the Jacobian transformation to $y = \frac{\gamma_i}{\bar{\gamma}_i}$ yielding

$$f_i(\gamma) = f_{\gamma_i}(\gamma\bar{\gamma}_i)\bar{\gamma}_i, \tag{15}$$

and the corresponding CDF $F_i(\cdot)$ is therefore given by

$$F_i(\gamma) = \int_0^\gamma f_i(x)dx = F_{\gamma_i}(\gamma\bar{\gamma}_i). \tag{16}$$

Substituting (15) and (16) into (14), one can get the PDF of the system output SNR as

$$f_{\gamma_s}(\gamma) = \sum_{i=1}^{K} f_{\gamma_i}(\gamma) \prod_{\substack{l=1 \\ l\neq i}}^{K} F_{\gamma_l}\left(\frac{\gamma}{\bar{\gamma}_i}\bar{\gamma}_l\right). \tag{17}$$

Again, substituting (17) into (9), one can get the capacity of the multiuser system employing the A/N hybrid scheduling.

B. Access Probability and Fairness

1) N/A Hybrid Scheduling: The access probability of the user j in the ith group for N/A hybrid scheduling scheme can be shown to be given by

$$\begin{aligned} P_{i,j} &= \Pr\left[\gamma_i = \gamma_{i,j} \text{ and } \gamma_i > \text{ all other } \gamma_n\right] \\ &= \frac{1}{N_i} \int_0^\infty f_{\gamma_i}(\gamma) \prod_{\substack{n=1 \\ n\neq i}}^{K} F_{\gamma_n}(\gamma) d\gamma. \end{aligned} \tag{18}$$

2) A/N Hybrid Scheduling: Similarly, the access probability of the user j in the ith group for A/N hybrid scheduling scheme can be shown to be given by

$$\begin{aligned} P_{i,j} &= \Pr\left[\gamma_i = \gamma_{i,j} \text{ and } \frac{\gamma_i}{\bar{\gamma}_i} > \text{ all other } \frac{\gamma_n}{\bar{\gamma}_n}\right] \\ &= \int_0^\infty f_i(\gamma) \prod_{\substack{n=1 \\ n\neq i}}^{K} F_n(\gamma) d\gamma \int_0^\infty f_{\gamma_{i,j}}(\gamma) \prod_{\substack{k=1 \\ k\neq j}}^{N_i} F_{\gamma_{i,k}}(\gamma) d\gamma, \end{aligned} \tag{19}$$

where $f_i(\cdot)$ is given in (15).

These access probabilities can be used to calculate the respective degree of fairness describe in Section II-C.

C. Grouping Issue and Applicability of the Hybrid Schemes

The grouping can be done in different ways. The system can divide the total active users based on their distances/areas to the base station, which we refer to as the natural grouping. A random grouping can be formed by a simple coin-tossing procedure.

Based on the grouping method employed by multiuser systems, the applicability of the proposed hybrid scheduling schemes can be classified to three categories.

1) Grouping users sector by sector:
   In this case, users within each group can have very disparate channels. In such case, it is beneficial to select users in the first step according to the normalized scheduling criterion. The final user will be selected based on the absolute scheduling criterion.
2) Grouping users ring by ring (based on their distance from the base station):
   In such case, users within each group have less disparate channel conditions. Therefore it is more reasonable to select users in the first step according to the absolute scheduling criterion and the second step is done according to the normalized scheduling criterion.
3) Random grouping:
   Grouping is done by randomly assigning users to K groups. In such case, there is no preference about which criterion should be used in the first step.

V. Numerical Examples

Fig. 1 and 2 plot the system average capacity in bps/Hz and system average fairness versus the number of users L for N/A and A/N hybrid scheduling schemes and various values of the number of groups K. The setup of Fig. 1 and 2 is as follows. Each time we increase the number of users by 2. The average SNR values of these two new users are generated from uniform$(0,1)$ and then normalized so that they add up to 2. Therefore the total SNR of L users is L. The same sets of random average SNR values are used by both Fig. 1 and 2 to see the capacity vs. fairness tradeoff. From Fig. 1, we see that the system average capacity for both the N/A and A/N hybrid scheduling schemes increases for larger L, as expected. Note that for the N/A hybrid scheduling the system average capacity also increases with the number of groups K. In particular, significant capacity boost can be seen when K changes from 1 to 2. On the other hand, the system average capacity for the A/N hybrid scheduling decreases as K increases. This is as expected by intuition since increasing K will make the N/A scheduling closer to the absolute scheduling and the A/N scheduling closer to the normalized scheduling. Fig. 2 gives the corresponding results for the system average fairness. From this figure one can clearly see that the system average fairness decreases for the N/A scheduling but increases for the A/N scheduling as K increases.

In figure 3 and 4, we show the effect of different number of users in each group on the scheduling gain. In the natural grouping case, we use the same number of users for each group. We generate users in these two figures based on more realistic models described in [8]. The average SNR $\bar{\gamma}$ is assumed to be log-normal distributed with a standard deviation of σ dB and a mean value according to the path loss model which is assumed to decrease exponentially with distance with an exponent denoted by α. The PDF of average SNR $\bar{\gamma}$ is based on

$$f_{\bar{\gamma}}(\bar{\gamma}) = \frac{1}{10\alpha\log e}\exp\left(\frac{2\sigma^2 - 2(\bar{\gamma}-\bar{\gamma}_R)10\alpha\log e}{100\alpha^2\log^2 e}\right) \times \left[1 - \mathrm{erf}\left(\frac{2\sigma^2 - (\bar{\gamma}-\bar{\gamma}_R)10\alpha\log e}{10\sqrt{2}\sigma\alpha\log e}\right)\right] \qquad (20)$$

The parameters used for simulations are given by $\alpha = 3.5$, $\sigma = 5$ dB, and $\bar{\gamma}_R = 12$ dB. One can see that having different numbers of users in each group has negligible impact on both the system capacity and fairness. Therefore the key factor of the proposed hybrid scheduling schemes is the number of groups while the employed grouping method is less important.

VI. Conclusion

This paper proposed a new hybrid multiuser scheduling scheme and analyzed its performance in terms of system throughput and the degree of fairness among users. It was shown that more flexible capacity vs. fairness tradeoffs can be made by grouping all the users in multiuser systems and scheduling the channel access in two steps. Different grouping methods and their effect on choosing the scheduling criteria were also discussed. Numerical examples were provided to demonstrate the benefits of using this hybrid scheduling.

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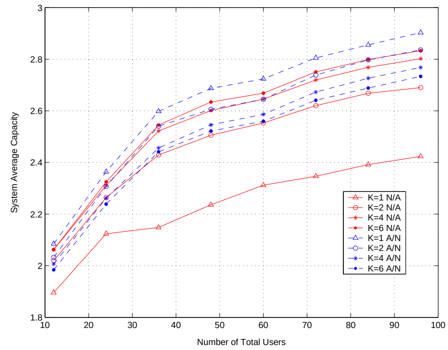

Fig. 1. Comparison of the system average capacity in bps/Hz versus the number of users L for the N/A and A/N hybrid scheduling schemes and various values of the number of groups K.

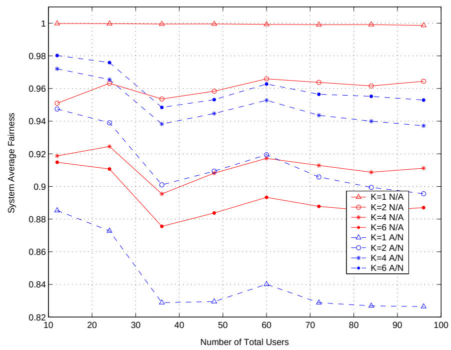

Fig. 2. Comparison of the system average fairness versus the number of users L for the N/A and A/N hybrid scheduling schemes and various values of the number of groups K.

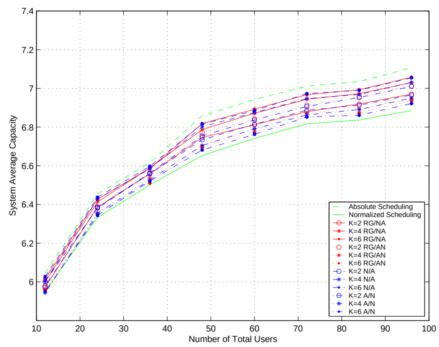

Fig. 3. Comparison of the system average capacity for the hybrid scheduling schemes employing random grouping and natural grouping with equal number of users in each group.

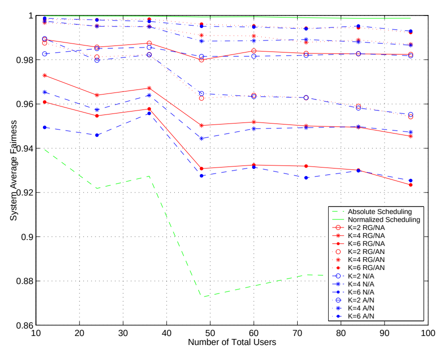

Fig. 4. Comparison of the system average fairness for the hybrid scheduling schemes employing random grouping and natural grouping with equal number of users in each group.