Effect of Antenna Correlation and Rician Fading on Capacity and Diversity Gains of Wireless MIMO Channels

Syed M. Tabish Qaseem, and Adel A. Ali

Department of Electrical Engineering, College of Engineering, King Saud University,

Riyadh, Saudi Arabia 11421

Abstract - **The use of antenna arrays at both sides of the wireless communication link can result in high channel capacity provided the propagation medium is rich scattering or Rayleigh fading and the antenna arrays at both sides are uncorrelated. However, the presence of LOS component and correlation of real world wireless channel may affect the system performance. In this paper, we investigate the effect of Rician factor (*K*) and the correlation coefficient (*r*) on the capacity and diversity of multi-input multi-output (MIMO) systems. We present the view point that the loss or gain in the capacity or diversity can be considered as an equivalent gain in the SNR.**

Index Terms- **Correlation, fading, multiple-input multiple-output (MIMO) systems, diversity, Rician channel, spatial multiplexing**

I. Introduction

Traditionally, multiple antennas have been used to increase diversity to combat channel fading. It is also widely understood that in a MIMO system the spectral efficiency (capacity) is much higher than that of the conventional single-antenna channels [1]–[2]. Hence, a MIMO system can provide two types of gains: diversity gain and spatial multiplexing gain or capacity gain. For radio systems with rich scattering and independently fading channels, with T transmit and R receives antennas, the system can provide spatial multiplexing gain of $m = min\ (T, R)$ or diversity gain (D) of order $T.R$ [3]. However, the correlation of a real-world wireless channel may result in a substantial degradation of the MIMO architecture performance [4]. Secondly, there is a possibility that the line-of-sight (LOS) component may exist in addition to scattered components. Then, the fading will follow the Rician distribution. The study of the capacity gains of correlated MIMO Rician channels is reported in [5] based on computer simulations. Recently, Sun and Reed [6] presented diversity analysis for MPSK transmitted over Rician fading channels, assuming that the fading channels are uncorrelated. However, the effect of correlated fading on diversity in Rician fading channels needs to be investigated.

In this paper we investigate the effect of Rician fading and correlation on the capacity and diversity of MIMO channels.

II. Overview of the Paper

The paper is organized as follows. Section III describes the signal and the channel model. The capacity and diversity gains of the Rician fading channel are discussed in section IV and section V, respectively. Section VI presents the simulation results and discussion followed by conclusions in section VII.

III. MIMO Channel and Signal Model

Consider a single user MIMO system with T antennas at the transmitter and R antennas at the receiver. For simplicity we consider only frequency flat fading; i.e., the fading is not frequency selective.

A. MIMO Capacity Signal Model

The system is described by the matrix equation:

$$\mathbf{y} = \sqrt{\frac{E_s}{T}}\mathbf{Hs} + \mathbf{n} \qquad (1)$$

where E_s is the total energy available at the transmitter, $\mathbf{y}$ is the R x 1 vector of signals received on the R antennas, $\mathbf{s}$ is the T x 1 vector of signals transmitted on the T transmit antennas, $\mathbf{n}$ is the R x 1 noise vector consisting of independent complex Gaussian distributed elements with zero mean and variance σ^2, and $\mathbf{H}$ is the R x T channel matrix.

B. MIMO Diversity Signal Model

Let there be D identical independent fading links exists between transmitter and receiver. Then, the signal model is given as:

$$y_i = \sqrt{\frac{E_s}{D}}h_i s + n_i\ , \quad i = 1,2,\cdots,D \qquad (2)$$

where y_i is the received signal on the i^{th} diversity branch, h_i is the channel corresponding to the i^{th} diversity branch, s is the transmitted symbol. E_s/D is the symbol energy available to the transmitter for each of the D branches and n_i is the AWGN with variance σ^2. In vector format, (2) can be written as:

$$\mathbf{y} = \sqrt{\frac{E_s}{D}}\mathbf{hs} + \mathbf{n} \qquad (3)$$

where $\mathbf{y}$ is the D x 1 vector of signals received on the R antennas, $\mathbf{s}$ is the T x 1 vector of signals transmitted on the T transmit antennas, $\mathbf{n}$ is the D x 1 noise vector consisting of independent complex Gaussian distributed elements with zero mean and variance σ^2, and $\mathbf{h}$ represents the vector form of $\mathbf{H}$ matrix, denoted as

$$\mathbf{h} = ved(\mathbf{H}) \qquad (4)$$

C. Correlated Rician fading Channel Model

For Rician fading the elements of $\mathbf{H}$ are non-zero mean complex Gaussians. Hence we can express $\mathbf{H}$ in matrix notation as [7]

$$\mathbf{H} = a\mathbf{H}^{sp} + b\mathbf{H}^{sc} \quad . \qquad (5)$$

where the specular and scattered components of $\mathbf{H}$ are denoted by superscripts sp and sc, respectively, $a > 0$, $b > 0$ and $a^2 + b^2 = 1$. $\mathbf{H}^{sp}$ is a matrix of unit entries denoted as $\mathbf{H}_1$. If there is no correlation at the transmitter or at the receiver side then the entries of $\mathbf{H}^{sc}$ are independent and identically distributed (i.i.d.) zero mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance, usually denoted by $\mathbf{H}_{\omega}$.

If there is correlated fading then the $\mathbf{H}^{sp}$ matrix can be modeled as [8]:

$$\mathbf{H}^{sp} = \mathbf{R}_{\mathbf{r}}^{1/2}\mathbf{H}_{\omega}\mathbf{R}_{\mathbf{t}}^{1/2} \qquad (6)$$

Where $\mathbf{R}_t$ and $\mathbf{R}_r$ are the correlation matrix at the transmitter and at the receiver side, respectively. The correlation matrix $\mathbf{R}$ is defined as [9]

$$r_{ij} = \begin{cases} r^{j-i}, & i \le j \\ r^{*}{}_{ji}, & i > j \end{cases} \quad , |r| \le 1 \qquad (7)$$

Where "*" denotes the complex conjugate. The Rician factor, K is defined as a^2/b^2. Thus, the above $\mathbf{H}$ matrix can be written as:

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}_1 + \sqrt{\frac{1}{K+1}}\mathbf{R}_{r}^{1/2}\mathbf{H}_{\omega}\mathbf{R}_{t}^{1/2} \qquad (8)$$

IV. MIMO CAPACITY

In the following, we assume that the channel is perfectly known to the receiver. Furthermore, we assume an ergodic block fading channel model where the channel remains constant over a block of consecutive symbols, and changes in an independent fashion across blocks.

A. Channel Unknown at the Transmitter

The MIMO channel capacity is given as [1], [2], and [10]

$$C = E_{\mathbf{H}}\left\{\sum_{i=1}^{k} \log_2\left(1 + \frac{E_s}{T\sigma^2}\lambda_i\right)\right\} \qquad (9)$$

Where $E_{\mathbf{H}}\{.\}$ denote the expectation over $\mathbf{H}$, the operator $\mathbf{H}^H$ indicates the hermitian of the matrix $\mathbf{H}$, k, ($k \le m$) is the rank of $\mathbf{H}$ and λ_i (i = 1, 2, ….., k) denotes the positive eigenvalues of $\mathbf{HH}^H$.

B. Channel Known at the Transmitter

With CSI being available at the transmitter, the capacity is given as [2].

$$C = E_{\mathbf{H}}\left\{\sum_{i=1}^{k} \log_2(\mu\lambda_i)^{+}\right\} \qquad (10)$$

Where μ is chosen to satisfy:

$$\frac{\rho}{N_o} = \sum_{i=1}^{k}\left(\mu - \lambda_i^{-1}\right)^{+} \qquad (11)$$

and "+" denotes taking only those terms which are positive.

V. MIMO DIVERSITY

In the following, we assume that the channel is perfectly known to the receiver. Furthermore, we assume an ergodic block fading channel model where the channel remains constant over a block of consecutive symbols, and changes in an independent fashion across blocks. Using maximal-ratio combining, the received signal (z) is given as

$$z = \mathbf{h}^H\mathbf{y} = \sqrt{\frac{E_s}{D}}\mathbf{h}^H\mathbf{h}s + \mathbf{h}^H\mathbf{n} \qquad (12)$$

Thus, signal-to-noise ratio (SNR) is

$$\eta = \frac{\frac{E_s}{D}\left(\|\mathbf{h}\|^2\right)^2}{\sigma^2\|\mathbf{h}\|^2} = \frac{1}{D}\|\mathbf{h}\|^2\bar{\gamma}_s \quad . \qquad (13)$$

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where $\|\mathbf{h}\|$ represents norm of vector **h**, and $\bar{\gamma}_s$ is the average SNR per symbol. The probability of symbol error for MPSK transmitted over an AWGN channel (non fading) is given as follows [11]:

$$P_s(E) = 2Q\left(\sqrt{2\gamma_s}\sin\frac{\pi}{M}\right) = 2Q\left(\sqrt{2k\gamma_b}\sin\frac{\pi}{M}\right) \quad (14)$$

where M is the signal alphabet, k is the number of bits per symbol, γ_s is SNR per symbol, γ_b is the SNR per bit, and $Q(\ )$ represents Gaussian Q-function. Thus, the probability of symbol error (SER) for MPSK for a fading channel can be obtained by modifying (14) as follows:

$$P_s(E) = 2Q\left(\sqrt{2\eta}\sin\frac{\pi}{M}\right) = 2Q\left(\sqrt{\frac{2\|\mathbf{h}\|^2\bar{\gamma}_s}{D}}\sin\frac{\pi}{M}\right)$$

$$= 2Q\left(\sqrt{\frac{2\|\mathbf{h}\|^2 k\bar{\gamma}_b}{D}}\sin\frac{\pi}{M}\right) \quad (15)$$

VI. SIMULATION RESULTS AND DISCUSSION

The results in this section illustrate the effect of antenna correlation and Rician fading on the capacity and diversity gains of MIMO channels. Here, in all cases we have considered correlation at both ends. This section is divided into two parts: the first part presents the capacity results while in the second part we discuss the diversity gains by calculating the probability of symbol error for Rician MIMO channels.

A. *Capacity Results*

To illustrate the effect of K factor and correlation (r) on the spatial multiplexing gain or capacity, we choose an m x m system. Here, the effect of correlation and K factor can be considered as an equivalent increase or decrease in the SNR for a given fixed capacity. The results are presented in table 1, and we can deduce the following:

1) For uncorrelated Rayleigh channels, capacity increase linearly with m or in other words for a given fixed capacity, SNR reduces linearly with m.
2) Increase in K factor reduces the capacity of MIMO system (except for m = 1). This is because the increase in K emphasizes the deterministic part of the channel. The deterministic channel is of rank 1 and so the capacity decreases.
3) With the increase in K, capacity increases for m = 1 because here the increase in K shifts the channel from Rayleigh fading towards Gaussian channel.
4) For a given value of K, the equivalent loss in SNR is more for higher value of m than for the lower values of m.
5) Increase in correlation is equivalent to the decrease in SNR i.e., with the increase in correlation, the capacity decreases. However, the equivalent loss in SNR corresponding to any value of correlation seems to be almost independent of m (e. g. r = 0.7 is equivalent to approximately 3 dB decrease in SNR).

TABLE I
EQUIVALENT GAIN IN SNR (dB),
C = 10 bits/sec/Hz T=R=m

m	K = 0 r = 0	K = 5 r =0	K = 10 r = 0	r = 0.7 K = 0	r = 0.9 K = 0
1	32.61	+1.73	+2.11	0	0
2	17.91	-1.79	-2.77	-2.63	-6.15
4	09.03	-3.39	-4.87	-3.13	-7.61
6	05.13	-3.96	-5.61	-2.95	-7.52

One way to reduce the loss in the capacity of MIMO systems due to Rician factor and correlation is to use CSI at the transmitter. Figure 1 depicts the percentage gains in capacity due to the availability of CSI at the transmitter for different values of SNR, K and r. The following can be deduced from the figure 1:

1) It is interesting to note that the water-filling gains are significant at low SNR and reduces at high SNR. The fact that water-filling gains are reduced at high SNR levels can be intuitively explained by the fact that knowledge of the CSI provides array gain both at the transmitter and at the receiver [10]. Since the relative importance of transmit array gain in boosting average SNR decreases in the high SNR region, the benefit of CSI at the transmitter also reduces.
2) Increasing the K factor reduces the capacity of the system as already explained. Therefore, when K increases, knowledge of CSI at the transmitter becomes relatively more important than when K is small. This trend is seen from the curves in Fig. 1-a
3) Increasing channel correlation (r) reduces the capacity of the system. Hence, for high correlation factor, knowledge of CSI at the transmitter becomes more important than when the correlation factor is small. This trend is seen from the curves in Fig. 1-b.

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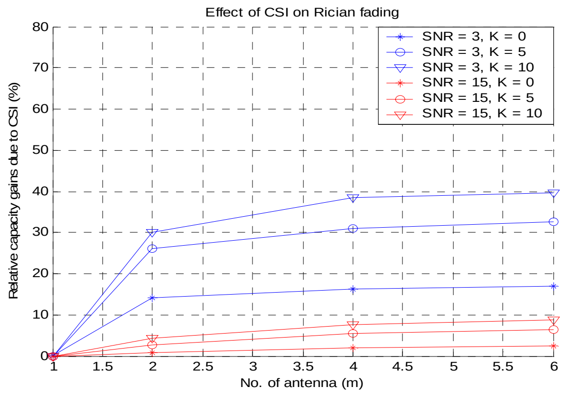
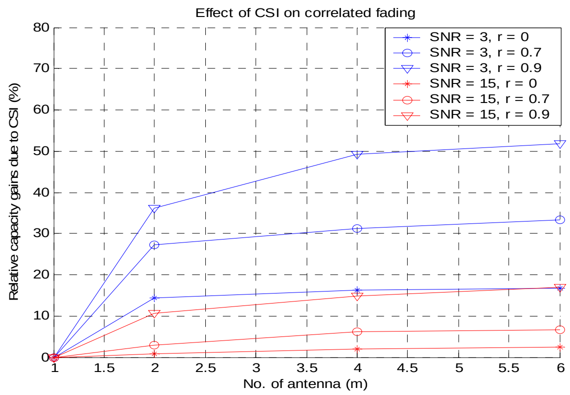


a) Effect of CSI with Rician factor (K), r = 0



b) Effect of CSI with correlation (r), K = 0

Fig. 1: Percentage relative gains in capacity due to the availability of CSI at the transmitter for different values of SNR, Rician factor K and Correlation parameter (r).

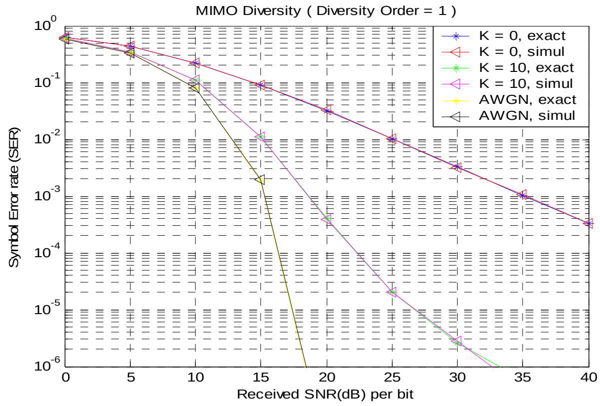
B. Diversity Results

For the validation of our model, we compare our simulated results with the exact results obtained by Sun and Reed [6]. The comparison is presented in figure 2. Then we introduce correlation and illustrate its effect on the diversity of MIMO Rician channels (presented in figure 3).

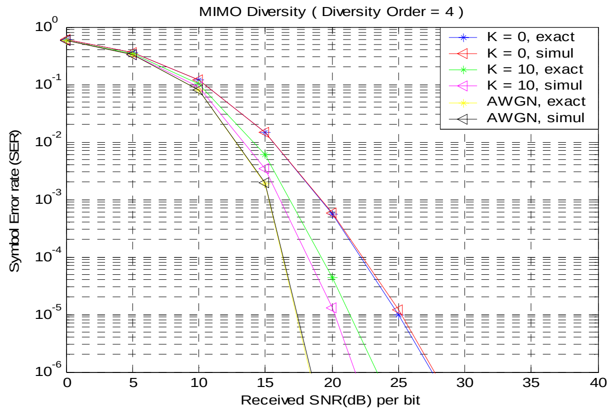
TABLE II

EQUIVALENT GAIN IN SNR (dB), $P_s(E) = 10^{-4}$

D	K = 0 r = 0	K = 5 r =0	K = 10 r = 0	r = 0.7 K = 0	r = 0.9 K = 0
1	45.15	+13.63	+22.44	0	0
2	29.25	+4.58	+8.36	-1.04	-3.54
4	23.02	+1.98	+3.07	-1.75	-5.38
6	20.08	+0.14	+0.26	-3.72	-7.26

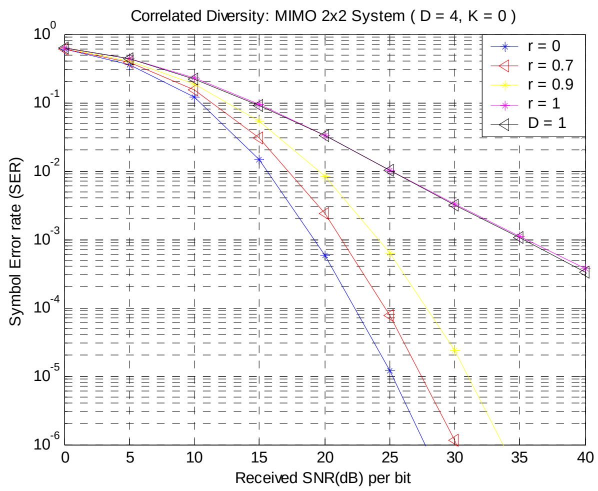


a) Diversity order (D) = 1, r = 0



b) Diversity order (D) = 4, r = 0

Fig. 2: Comparison of SER obtained from simulation with exact [6] for 16PSK over uncorrelated Rician fading channels for different diversity orders (D)



a) Diversity order = 4, K = 0

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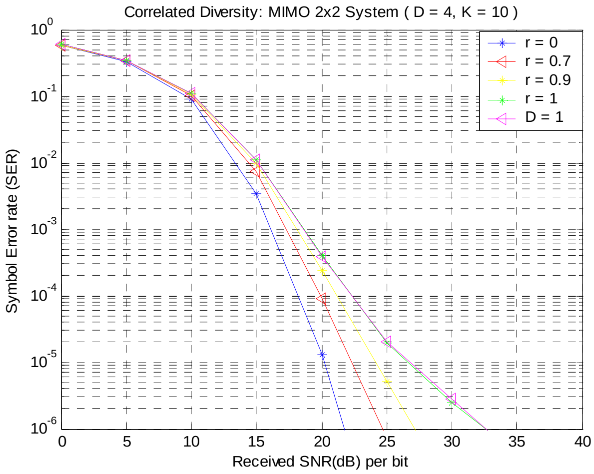

b) Diversity order = 4, K = 10

Fig. 3: SER for 16PSK over correlated Rician fading channels for Diversity order = 4.

From figure 2, it is clear that simulated results and exact results are almost same thus validating our approach. However, simulated results comply very well with lower values of D than with higher values of D. From figure 3, we deduce the following:

1) The correlation reduces the diversity order thereby increasing the symbol error rate. Correlation up to 0.7 is tolerable as it does not drastically affect the overall system performance. Correlation between 0.9 and 1.0 drastically affect the system performance.
2) Perfect correlation (correlation of magnitude 1.0) nullifies the affect of diversity.
3) Fading channels with higher values of K are less affected by correlated fading.

In table 2, we showed that the effect of correlation and K factor on the diversity gain can be considered as the equivalent increase or decrease in the SNR for a given fixed SER. From table 2, we deduce the following:

1) Increase in K factor improves the SER. This is because the increase in K factor stabilizes the link.
2) The improvement due to K factor reduces for higher diversity order.
3) Increase in correlation is equivalent to the decrease in SNR. However, the equivalent loss in SNR up to correlation of 0.7 is tolerable.

VII. Conclusion

We have investigated the effect of Rician fading and antenna correlation on the capacity and diversity of multi-input multi-output (MIMO) systems. We used computer simulation and presented the view point that the loss or gain in the capacity or diversity can be considered as an equivalent dB gain, positive or negative, in the SNR required for a given capacity or error rate performance.

The essence of the results is that correlation reduces the capacity and diversity gains, for large values of the correlation parameter r larger than 0.5. Smaller correlation has no serious effect on capacity and diversity gains whereas Rician fading reduces the capacity gains but increase the diversity gains of the MIMO system, in comparison to fully scattering Rayleigh fading. The loss or gain in the capacity or diversity can be considered as an equivalent increase or decrease in the SNR.

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