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PERFORMANCE ANALYSIS OF MQAM MODULATION SCHEMES WITH DIVERSITY IN NAKAGAMI FADING CHANNELS

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ABSTRACT

The performance of M-ary Quadrature Amplitude Modulation (MQAM) scheme is analyzed with different diversity schemes over Nakagami fading channels. The performance expressions are derived in closed form for this versatile fading channel model. Our final expressions do not involve numerical integration which is an advantage over the previously reported work. Furthermore, no accuracy is sacrificed.

Keywords: *Wireless, MQAM, Diversity, Nakagami Fading.*

الملخص

يقدم هذا البحث تحليلاً لأداء نظام MQAM عند تكرير إرسالها على القنوات الذاوية التي تتبع نموذج Nakagami. يعرض هذا البحث النتائج على صورة تعابير رياضية محددة لقياس الأداء، وهو ما يعتبر إضافة في هذا المجال على الأبحاث السابقة التي تعتمد في قياس الأداء على إجراء عمليات تكامل حسابية. تجدر الإشارة إلى أن التعابير الرياضية المشتقة دقيقة وليست تقريبية .

1. INTRODUCTION

To improve the reliability of communication on wireless channels, diversity techniques have to be employed to combat the detrimental consequences of fading. The idea behind diversity schemes is to extract information from signals that are received via multiple fading paths to improve the received signal-to-noise ratio (SNR).

Performance analysis for spectrally effient modulation schemes has not been considered thoroughly for Nakagami fading in diversity receivers, although Nakagami statistics is known to best fit experimental data of mobile channels [Nakagami, 1960].

This work considers spectrally efficient MQAM scheme with maximum ratio (MRC), equal gain (EGC) and selection (SC) combining diversity schemes in Nakagami fading. We found a concurrent but related work in [Annamalai et. al, 1999]. In contrast, their error probability expression involves integral evaluation, which can be computationally demanding. Our closed-form expression does not involve numerical integration. Also, non of the previous work on Nakagami and MQAM considered selection diversity. In the following, the performance analysis is given, followed by some numerical examples and finally conclusions are drawn.

2. PEFORMANCE ANALYSIS

The conditional error probability of M-QAM can be expressed as [Proakis, 1989].

$$P(e\,|\,\gamma) = 4(1-\frac{1}{\sqrt{M}})Q(\sqrt{2g\gamma}) - 4(1-\frac{1}{\sqrt{M}})^2 Q^2(\sqrt{2g\gamma}) \tag{1}$$

where $g = \dfrac{3}{2(M-1)}$.

The average symbol error rate, $P_{MQAM}(e)$ of M-QAM is obtained by averaging the conditional $P(e\,|\,\gamma)$ over the pdf $p(\gamma)$ of the average SNR, γ. This pdf can be expressed for MRC, EGC or SC diversity scheme. The error rate is subsequently expressed as,

$$P_{MQAM}(e) = 4(1-\frac{1}{\sqrt{M}})\underbrace{\int_0^\infty p(\gamma)Q(\sqrt{2g\gamma})d\gamma}_{I_1} - 4(1-\frac{1}{\sqrt{M}})\underbrace{\int_0^\infty p(\gamma)Q^2(\sqrt{2g\gamma})d\gamma}_{I_2} \tag{2}$$

2.1 Maximal Ratio Combining

Analysis of QAM in Nakagami Fading Channels with MRC Diversity is carried out using the pdf of the combined SNR of MRC that is expressed in the form,

$$p(\gamma) = \left(\frac{m}{\bar{\gamma}}\right)^{Lm} \frac{\gamma^{Lm-1}}{\Gamma(Lm)} \exp(-\frac{m\gamma}{\bar{\gamma}}) \tag{3}$$

Thus, we show that the average symbol error rate of M-QAM with MRC in Nakagami fading can be evaluated by expressing the integral I_1 of (2) as,

$$I_1 = \left[\frac{1}{2}\left(1-\sqrt{\frac{g\bar{\gamma}}{m+g\bar{\gamma}}}\right)\right]^{Lm} \sum_{k=0}^{Lm-1} \binom{Lm-1+k}{k} \left[\frac{1}{2}\left(1+\sqrt{\frac{g\bar{\gamma}}{m+g\bar{\gamma}}}\right)\right]^k \tag{4}$$

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Furthermore, we represent the $Q^2(.)$ function in a form that makes us able to evaluate the integral I_2 in a closed form. With this approach we are able to avoid numerical integration in the final error performance expressions, which is not the case with the previous work [Annamalai et. al, 1999]. The derived expression for I_2 is,

$$I_2 = \frac{1}{4} - \beta\left(\frac{1}{2} - \frac{\tan^{-1}(\alpha_2)}{\pi}\right)\sum_{k=0}^{Lm-1}\binom{2k}{k}\frac{1}{[4(\mu^2+1)]^k}$$

$$- \beta\frac{\sin(\tan^{-1}(\alpha_2))}{\pi}\sum_{k-1}^{Lm-1}\sum_{i=1}^{k}\frac{T_{ik}}{(\mu^2+1)^k}[\cos(\tan^{-1}\alpha_2)]^{2(k-i)+1} \tag{5}$$

where

$$\mu = \sqrt{\frac{g\bar{\gamma}}{m}},\ \alpha_2 = \sqrt{\frac{g\bar{\gamma}}{m+g\bar{\gamma}}},\ \beta = \sqrt{\frac{g\bar{\gamma}}{m+g\bar{\gamma}}},\ T_{ik} = \frac{\binom{2k}{k}}{\binom{2(k-i)}{(k-i)}4^i(2(k-i)+1)}$$

Putting I_1 (4) and I_2 (5) in (2), the performance of QAM in Nakagami Fading Channels with EGC can be expressed in closed forms.

2.2 For Equal Gain Combining

The same procedure can be followed for equal gain combining. We need only find the I_1 and I_2 to be used in (2) using the appropriate pdf as shown in [Al-Shalan, 2000]. The derived expressions are:

$$I_1 = \left[\frac{1}{2}\left(1 - \sqrt{\frac{g\bar{\gamma}_{egc}}{Lm+g\bar{\gamma}_{egc}}}\right)\right]^{Lm}\sum_{k=0}^{Lm-1}\binom{Lm-1+k}{k}\left(\frac{1}{2}\left[1+\sqrt{\frac{g\bar{\gamma}_{egc}}{Lm+g\bar{\gamma}_{egc}}}\right]^k\right) \tag{6}$$

and

$$I_2 = \frac{1}{4} - \beta\left(\frac{1}{2} - \frac{\tan^{-1}(\alpha_2)}{\pi}\right)\sum_{k=0}^{Lm-1}\binom{2k}{k}\frac{1}{\left[4(\mu^2+1)\right]^k}$$

$$- \beta\frac{\sin(\tan^{-1}(\alpha_2))}{\pi}\sum_{k=1}^{Lm-1}\sum_{i=1}^{k}\frac{T_{ik}}{(\mu^2+1)^k}\left[\cos(\tan^{-1}\alpha_2)\right]^{2(k-i)+1} \tag{7}$$

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where the following are defined,

$$\mu = \sqrt{\frac{g\bar{\gamma}_{egc}}{Lm}}, \alpha_2 = \sqrt{\frac{g\bar{\gamma}_{egc}}{Lm + g\bar{\gamma}_{egc}}}, \beta = \sqrt{\frac{g\bar{\gamma}_{egc}}{Lm + g\bar{\gamma}_{egc}}}, and \;\; T_{ik} = \frac{\binom{2k}{k}}{\binom{2(k-i)}{(k-i)} 4^i (2(k-i)+1)}$$

2.3 For Selection Combining

Similarly for selection combining we obtain:

$$I_1 = \frac{L}{\Gamma(m)} \sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} \sum_{k=0}^{l(m-1)} b_k^l \frac{\Gamma(m+k)}{(l+1)^{m+k}} \left[P_l(c)\right]^{m+k} \sum_{h=0}^{m=k-1} \binom{m+k-1+h}{h} \left[1 - P_l(c)\right]^h \quad (8)$$

where $P_l(c) = \frac{1}{2}(1-\mu_l)$ and $\mu_l = \sqrt{\frac{g\bar{\gamma}}{g\bar{\gamma} + m(l+1)}}$

and

$$I_2 = \frac{L}{\Gamma(m)} \sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} \sum_{k=0}^{l(m-1)} b_k^l \frac{\Gamma(m+k)}{(l+1)^{m+k}} [\tfrac{1}{4} - \beta_l \{ (\tfrac{1}{2} - \frac{\tan^{-1}(\alpha_l)}{\pi})$$

$$\sum_{h=0}^{m+k-l} \binom{2h}{h} \frac{1}{[4(\mu_l^2+1)]^h}$$

$$- \frac{\sin(\tan^{-1}(\alpha_l))}{\pi} \sum_{h=1}^{m+k-1} \sum_{i=1}^{h} \frac{T_{ih}}{(\mu_l^2+1)^h} [\cos(\tan^{-1}\alpha_l)]^{2(h-i)+l} \}] \quad (9)$$

with $\alpha_l = \beta_l = \sqrt{\frac{g\bar{\gamma}}{g\bar{\gamma} + m(l+1)}}$ and $\mu_l = \sqrt{\frac{g\bar{\gamma}}{m(l+1)}}$

3. NUMERICAL EXAMPLES

The analytical expressions and simulation results [designated with (O)] are plotted for 16-QAM in Figure 1 for the MRC, EGC, and SC for various diversity orders, L and Nakagami parameters, m. The Figure gives the performance curves for $m = 2$ unless otherwise indicated close to a curve.

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The plots indicate that perfect agreement exist between the simulation and derived expressions. A comprehensive result for other constellations (eg. 32-QAM, 64-QAM) and various Nakagami fading with parameter is reported in [Al-Shalan, 2000]. The effect of increasing the number of diversity branches was also compared for all the diversity schemes. The results of Rayleigh fading, Nakagami fading with parameter. (m=1), and non-diversity cases L=1 were also verified. Furthermore, simulation results are also reported for the emerging and promising hybrid of MRC and SC called generalized selection combining for this spectrally efficient modulation (MQAM) schemes.

CONCLUSION

In conclusion, closed form performance expressions have been derived for MRC, EGC, and SC receivers with MQAM modulation scheme over Nakagami fading channels. Existing expressions use numerical integration to get the error probability. Using our expressions, numerical integration can be avoided. Computer simulations were carried out and found to be in perfect agreement with analytical results.

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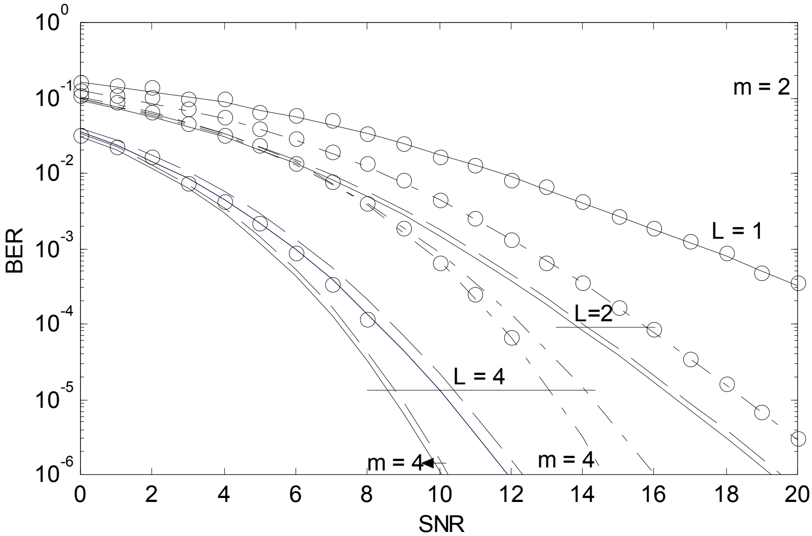

Figure 1: Performance of 16-QAM with diversity combining in Nakagami fading

(MRC: ___, EGC: - - -, SC: -.-.-., Simulation: o).

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