

# **EVALUATION OF ENERGY LOSSES IN PIPES**

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#### **ABSTRACT**

Energy losses in pipes used for the transportation of fluids (water, petroleum etc.) are essentially due to friction, as well as to the diverse singularities encountered. These losses are usually converted into head reductions in the direction of the flow. The knowledge of data of such transformation allows the determination of the necessary power needed for the transportation of the fluid between two points. It constitutes the necessary calculation basis necessary for the design and analysis of transport and distribution networks. The review of the different relationships allowing the determination of these losses and their comparison to the experimental results obtained by the authors constitute the object of this study.

**Keywords:** Head losses, Frictional head losses, Major head losses, Minor head losses, Singular head losses, Water distribution network.

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#### **NOTATIONS**

a,m,n: Calmon-Lechapt coefficients  $C_{HW}$ : Hazen-Williams coefficients

C<sub>s</sub> : Scobey coefficientD : Pipe diameter (m)

f : Pipe frictional coefficient

g : Gravitational acceleration  $(m/s^2)$ 

 $h_f$ : Pipe frictional head loss (m)

 $h_m$ : Pipe singular head loss (m)

*j* : Head loss per unit length (m/m)

K : Strickler coefficient

*Ka* : Karman number  $(R_e \sqrt{f})$ 

K... : Singularity coefficient

L: Pipe length (m)

N : Manning roughness coefficient

Q : Fluid flow rate  $(m^3/s)$ 

R: Radius (m)

 $R_e$ : Reynolds number

V : Pipe mean flow velocity (m/s)

 $\varepsilon$ : Pipe roughness

 $\Phi$  : Function

#### 1. INTRODUCTION

Pipe technology is based on the universal principles of fluid flow. When a real (viscous) fluid flows through a pipe, part of its energy is spent through maintaining the flow. Due to internal friction and turbulence, this energy is converted into thermal energy. Such a conversion leads to the expression of the energy loss in terms of the fluid height termed as the head loss and usually classified into two categories. Essentially due to friction, the first type is called linear or major head loss. It is present throughout the length of the pipe. The second category called minor or singular head loss is due to the minor appurtenances and accessories present in a pipe network. The appurtenance encountered by the fluid flow which is a sudden or gradual change of the boundaries results in a change in magnitude, direction or distribution of the velocity of the flow. This classification into major and minor head losses is rather relative. For a pipeline of small length having many minor appurtenances, the total minor head loss can be greater than the frictional head loss. In petroleum and water distribution networks, the pipelines are of considerable length and therefore the terms major head loss and minor head loss can be used without any confusion.

A great number of studies were carried out in order to achieve a general and precise formulation of the diverse types of head losses. [Weisbach, 1855] was the first to have come out with a relation for the head loss. As brought up by [Bhave, 1991], Darcy contributed greatly to the application of the derived relation, thus associating his name with that of

Weisbach. The relation is therefore most commonly known as the Darcy-Weisbach formula. It essentially depends on the friction coefficient and the relative roughness.

The friction coefficient is a function of the flow regime characterised by the Reynolds number. Several explicit and implicit relationships were proposed for the friction coefficient. [Nikuradse, 1933] performed extensive experimentations involving smooth and artificially roughened pipes achieved using sand particles of uniform size. The Nikuradse diagram also known as the Stanton diagram or the Stanton-Pannel diagram, is the result of these investigations. Comparing the results included in Nikuradse's diagram, [Colebrook, 1939] found that its curves do not match with those of actual pipes. However, by introducing the concept of equivalent surface roughness, it is possible to use Nikuradse's results for commercial pipes.

Several other investigators provided the literature with diverse diagrams. [Johnson, 1934] presented a diagram for commercial pipes using several non-dimensional groups. [Rouse, 1943] plotted the friction coefficient represented by  $(1/\sqrt{f})$  against the Karman number

represented by:  $K = R_e \sqrt{f} = D^{\frac{3}{2}} \sqrt{2g\frac{S}{v}}$ . He produced the necessary curves for the Colebrook

transition zone. L. F. Moody suggested to Rouse converting his diagram by plotting the friction coefficient against the Reynolds number which he refused. [Moody, 1944] then plotted it himself, producing the actual universally known Moody diagram allowing the determination of the friction coefficient as a function of the Reynolds number and the ratio  $(\varepsilon/D)$ .

Other investigators provided the literature with relationships. [Blasius, 1913] suggested a relationship between the friction coefficient and the Reynolds number applicable solely for smooth turbulent flows. For the same regime, [Ger and Holley, 1976] suggested a relationship similar to that of Blasius. For a transitional turbulent flow, [Colebrook, 1939] derived a relationship which is presently commonly known as the Colebrook-White equation. For a rough turbulent flow, Prandtl [Bhave, 1991] suggested a relationship expressing the friction coefficient as a function of the ratio  $(\varepsilon/D)$ . It is now widely known as the Karman-Prandtl equation.

All these relationships are implicit implying the use of a try-and-error procedure in order to achieve a solution leading to a value of the friction coefficient. Explicit relations expressing the friction coefficient for all regimes of flow are available. For a smooth turbulent flow, [Techo et al, 1965] suggested a simple explicit relation based on the Karman-Prandtl equation while [Chen, 1984] proposed a simpler one. For a transitional turbulent flow, [Moody, 1944] enriched the specialised literature with a highly accurate relationship, on which the relations of [Wood, 1972], [Barr, 1975], [Jain, 1976], [Swamee and Jain, 1976], [Zigrang and Sylvester, 1982], [Haaland, 1983] and [Chen, 1984] linking the friction coefficient f to the

Reynolds number  $R_e$  and the relative pipe roughness  $(\varepsilon/D)$  have been based and led to an appreciable accuracy.

Relationships applicable for all regimes are also available. [Churchill, 1977] proposed a friction factor equation which should be applicable for all fluid flow regimes whereas [Chen, 1985] derived explicit solutions for the Prandtl and Colebrook-White equations. One of the widely used formulas used is that developed by [Williams and Hazen, 1933]. It expresses the head loss as a function of diameter, flow rate and length using an empirical coefficient. The Hazen-Williams coefficient as it has since been known depends on the pipe material and the flow velocity. Values of the Hazen-Williams coefficient applicable for diverse common pipe materials are recommended by [Lamont, 1969] who tabled them after performing a great amount of experimentations. The Hazen-Williams formula was a subject of interest for many investigators. Based on his own tests, [White, 1974] confirmed it while [Janna et al, 1978] tried to modify it by introducing a new coefficient which vary with the Reynolds number, the relative roughness and the flow velocity. Earlier in 1923, as reported by [Morel and Laborde, 1994] Strickler had suggested a simpler relationship based on a fixed coefficient. Later on, other empirical relationships were proposed. Hence, [Scobey, 1966] suggested his relationship as early as 1930. It is similar to that of Hazen-Williams except for the use of a fixed coefficient. In 1965 as reported by [Morel and Laborde, 1994], Calmon and Lechapt, suggested a relationship including three coefficients which vary with the pipe roughness. The Manning formula known as such because it has firstly been derived by Manning in 1891 [Bhave, 1991] has, as reported by [Powell, 1968] and [Williams, 1970], since received a lot of contributions and is still being used.

The various experimentations carried out by the authors in order to determine the linear and singular head losses taking place in a variety of pipes of different dimensions and roughness are presented. Water is used at different flow regimes. The obtained linear results are compared to those computed using the most common pipe flow formulas presented earlier (Darcy-Weisback, Hazen-Williams, Manning, Strickler, Scobey and Calmon & Lechapt) leading to the domain of application and accuracy of these relationships.

Eight singularities widely used in distribution networks have also been investigated. They include sudden enlargements and contractions, 45° and 90° bends of different radiuses of curvature, Venturi meters, orifice meters, gate valves and ball valves. The results obtained are compared to those available in the literature.

### 2. EXPERIMENTAL METHOD

The experimental investigations were carried out on a test bench mainly constituted by a hydraulic bench providing the necessary flow discharge, a network of pipes of different

dimensions and roughness able to simulate both frictional (major) and singular (minor) head losses.

The hydraulic bench used to distribute the water at the required flow rates, is principally constituted by a reservoir and two pumps which may be used in series or in parallel [Boudebza and Bouachari, 1999]. The water is pumped from reservoir to the pipes network through a closed circuit. A regulation valve, a direct rotameter, and water and mercury manometers are used to regulate the fluid flow and to determine the pressure in terms of head difference respectively.

The network is constituted by PVC pipes of different diameters and roughness. Four are pipes used for frictional head losses. Their diameter vary from 13mm to 25mm, and their surface roughness from smooth to  $\varepsilon = 0.02mm$ . The network of conduits also includes diverse appurtenances simulating singular head losses. It includes a Venturi meter, an orifice meter, a 45° bend, a ball valve, a sudden enlargement, a sudden contraction, two 90° bends of large and small radius of curvature respectively, a strainer, a gate valve and an open valve.

Measurements are carried out in terms of height using water and mercury manometers. They represent the difference of pressure between the respective positions where they are realized. The necessary flow discharges are obtained through two centrifugal pumps, and the flow rate is insured by a rotameter.

#### 3. THEORETICAL APPROACH

### 3.1. Frictional or major head losses

Frictional head losses are mainly due to the fluid viscosity and the flow regime. Their influence may be resented throughout the length of the pipe. In a long pipe, the frictional head losses are relatively important, and they cannot be neglected. A relationship expressing this loss is proposed by [Weisbach, 1855]. Known as the Darcy-Weisbach equation, it links the head loss the friction coefficient, the flow velocity and the pipe dimensions:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{1}$$

By expressing the velocity as a function of flow pipe section  $(V = 4Q/\pi D^2)$  and by replacing the known parameters by their respective numerous values, equation (1) becomes:

$$j = \frac{f}{12.1 D^5} Q^2 \tag{2}$$

Dimensional analysis of the Darcy-Weisbach equation leads to:

$$h_f = \frac{LV^2}{Dg} \phi \left[ \frac{\varepsilon}{D}, R_e \right]$$
 (3)

Comparing equations (1) and (3) shows that the Darcy friction coefficient f is a function of both relative roughness and Reynolds number. [Nikuradse, 1933] led the way in trying to express the friction coefficient by carrying out extensive experimentations leading to the measurement of the velocity distribution and head losses throughout the length of smooth and artificially roughened pipes. He applied the well-known Prandtl mixing length theory, and for smooth turbulent flow, developed a relationship for f which is unfortunately of implicit type requiring a recursive solution. For such a flow filed and for values of the Reynolds number less than  $10^5$ , [Blasius, 1913] reached a much more simple relation:

$$f = \frac{0.316}{R_{\circ}^{0.25}} \tag{4}$$

However, almost all pipe flow regimes correspond to a transition zone where the frictional coefficient depends both on the Reynolds number and the relative roughness. [Colebrook, 1939] suggested a relationship in this direction since termed the Colebrook-White equation. It also had an implicit form, but was interesting since almost all the explicit solutions proposed later are approximations of this relation. [Zigrang and Silvester, 1982] derived a complex relationship which has the advantage of being explicit and highly accurate for  $4x10^3 \le R_e \le 10^8$  and  $0 \le \varepsilon / D \le 5x10^{-2}$ :

$$\frac{1}{\sqrt{f}} = -2\log\left\{\frac{\varepsilon}{3.7D} - \frac{5.02}{R_e}\log\left[\frac{\varepsilon}{3.7D} - \frac{5.02}{R_e}\log\left(\frac{\varepsilon}{3.7D} + \frac{13}{R_e}\right)\right]\right\}$$
(5)

Other empirical relationships proposed later have the merit of expressing the head loss without the complexity introduced by the friction coefficient. They are mostly based on polynomial representation. Mostly used in the U.S.A., the relationship developed by [Williams and Hazen, 1933] may be written as:

$$j = \frac{10.68}{C_{HW}^{1.852} D^{4.87}} Q^{1.852}$$
 (6)

The value of the Hazen-Williams coefficient C<sub>HW</sub> is a function of the material and its dimensions. Its value is found to be 145, 146, 148 and 150 for PVC-roughened-14mm-diameter pipes, PVC-roughened-25mm-diameter pipes, PVC-smooth-13.3mm-diameter pipes, and PVC-smooth-23.5mm-diameter pipes respectively [Bhave, 1991].

Another often used relationship is that known as the Manning equation, although its attribution to this later is contested as reported by [Powell, 1968] and [Williams, 1970]. It is expressed as:

$$j = \frac{10.29 \, N^2}{D^{16/3}} \, Q^2 \tag{7}$$

The Manning roughness coefficient is solely dependant on the pipe material. Its value is found to be 0.0095 for the type of PVC used in the experimentations [Bhave, 1991].

The other relationships investigated in this study are those due to:

Strickler [Morel and Laborde, 1994]:

$$j = \frac{10.29}{K^2 D^{16/3}} Q^2 \tag{8}$$

[Scobey, 1966]:

$$j = \frac{1.58}{C_s^{1.886} D^{4.87}} Q^{1.887}$$
 (9)

Calmon and Lechapt [Morel and Laborde, 1994]:

$$j = a \frac{Q^n}{D^m} \tag{10}$$

where:

K = 0.95 for the type of pipes and fittings used.

 $C_s = 37$  for the new pipes used.

 $a = 0.916.10^{-3}$ ; n = 1.78; m = 4.78 for the smooth pipes used.

 $a = 1.01.10^{-3}$ ; n = 1.84; m = 4.88 for the roughned pipes used.

## 3.2. Singular Or Minor Head Losses

Head losses due to singularities or accessories are commonly termed minor head losses. This is due to the fact that for pipes of important length, their value can be neglected comparatively to that due to friction. However, their effect can be significant for short pipes. Minor head losses are expressed as:

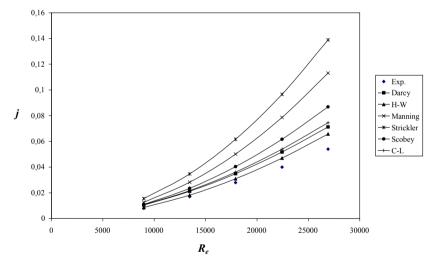
$$h_m = K_m \frac{V^2}{2g} = \frac{8K_m}{\pi^2 g D^4} Q^2 \tag{11}$$

The constant  $K_m$  is a characteristic of the type of singularity, and its value varies consequently.

#### 4. RESULTS AND COMPARISON

Figures 1 to 4 present the results obtained concerning the frictional head losses for the four pipes investigated. They are presented in terms of the linear head loss  $(h_f/L)$  versus the Reynolds number. All flow regimes were found to be turbulent.

For the two smooth pipes for which the results are presented in figures 1 and 2, the Hazen-Williams and Darcy approaches seem to be the more able to predicting the head loss. The integration of complex expressions for the friction coefficient with their large spectrum of application, like the one expressed in equation (5) [Zigrang and Silvester, 1982], seems to be appropriate and ends up with satisfactory results.



**Figure 1:** Head loss for a smooth pipe (D=23.5mm)

It is nevertheless remarkable that the increase in the Reynolds number leads to higher errors. This is probably due to the fact that the flow field moves from the laminar/transitional regime to the turbulent regime. This difference can be corrected by integrating appropriate coefficients considering the change of regime, and this shows the complexity and the sensibility of the problem particularly when it is applied for the solution of distribution networks which may contain a great number of pipes with distinct diameters and roughness [Haddad and Lahiouel,

1999]. This problem is not met when one deals with the energy losses encountered in pipelines transporting petroleum since these have usually the same characteristics [Haddad et al., 1999].

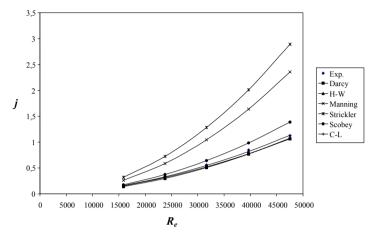
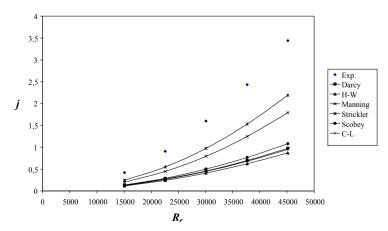
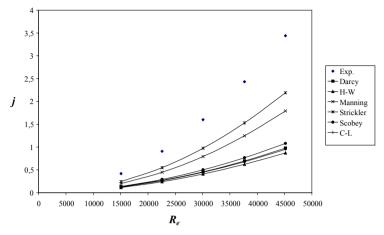


Figure 2: Head loss for a smooth pipe (D=13.3mm)

The same comments can be made for the case of the roughened pipes for which the results are presented in figures 3 and 4. The experimental results show head losses greater than those computed by the different empirical formulas used. The authors comment on this difference by the fact that the roughness coefficient which is the most important parameter might have not been suitably predicted. Its value has in fact been chosen equal to that suggested by the manufacturer, thus neglecting the possible presence of deposits which generally tends to increase the roughness of pipe walls and consequently the head loss.



**Figure 3:** Head loss for a roughened pipe  $(D = 25 \text{ mm} - \varepsilon = 0.015 \text{ mm})$ 



**Figure 4:** Head loss for a roughened pipe  $(D=14mm-\varepsilon=0.02mm)$ 

The capacity of transportation of pipelines decreases with time (age). This loss is mainly caused either by a diminution in the cross section area due to an accumulation of deposits inside the pipe, or an increase of the ruggedness, or both. The biological growth, the obstruction and the encrustation are the most common forms of such deposits which can vary from 1mm to 10mm in thickness. For the Hazen-Williams formula, the value of 140 is the most commonly used for  $C_{HW}$ . It is however understood that for an old pipe in good condition, a value comprised in the interval 100-120 should be acceptable, while for a used pipe 40-80 is mostly used. This situation shows the great difficulties faced by the authors in achieving an 'acceptable' value for the coefficients ( $C_{HW}$  as well as f and N) used in the developed relationships.

 Table 1: Head loss for singularities

	Sudden	Sudden	Sudden	Progressive	Venturi
	enlargement	Contraction	Bend (90°)	Bend (45°)	meter
	$\int D_{upstream} = 13.3mm$	$\int D_{upstream} = 23.5mm$			
	$D_{downstream} = 23.5mm$	$D_{downstream} = 13.3mm$	$R_{C}$ - median	-Long radius-	-short pipe-
	0.01	0.20	0.70	0.40=	0.61
Experimental	0.21	0.39	0.70	0.197	0.61
results	$V = 0.6m/s \div 13m/s$ $D = 1 \div 1.8$	$V = 0.6m/s \div 13m/s$ $D = 1.8 \div 1$	$V = 0.6m/s \div 13m/s$	with edge	R: 1 ÷ 2
Empirical results [Bhave, 1991]	0.40	0.34	0.75	0.18	0.72

The results obtained through the different singularities investigated are resumed in table 1. Great difficulties have been faced by the authors when dealing with minor losses. Indeed, the measurements through such geometries have been found to be complex and, the literature dealing with such phenomenon is poor compared to that interested in the friction losses. In the case of a sudden enlargement for example, the re-establishment of the velocity field takes a distance of approximatively 100 the pipe diameter. Within this intermediate region, the flow is complex involving both friction and turbulence, and it is difficult to separate the effects of the latter from that due to friction. Furthermore, the results published suggest values for the minor head loss coefficients generally neglecting important parameters such as material which may have a great influence upon the result.

The results proposed for the minor losses seem to be acceptable, except in the case of the sudden enlargement where there is a clear divergence. Te authors feel that more interest should be given to this kind of local losses, and that future investigations should try to take into account all the factors involved in such head losses.

#### 5. CONCLUSIONS

The present study investigated the different approaches developed for the determination of energy losses inside fluid transportation pipes. These losses may be caused either by friction or appurtenances.

The experimental results obtained and their comparison to those computed using the relationships developed by diverse investigators over the last decades show a preponderance of the relations proposed by Darcy and Hazen-Williams. The errors found seem to be mainly due to the difficulty to determining the friction coefficient which is a function of the Reynolds number and wall pipe roughness. The main difficulty which arises when trying to determine the pipe roughness is due to the fact that such pipes are subject to age effect resulting in erosion, corrosion, deposits, etc. The coefficient value proposed by the manufacturer is no more valid. Further research might well then be devoted to the phenomenon of aging of pipes.

More complexity is faced when head losses through singularities are investigated. The intermediate region downstream of any appurtenance is a mixture of friction and turbulence phenomena, and it is difficult to separate the effects of each one. Further research should be directed towards defining precise minor coefficients by taking into account all the factors involved in such head losses.

The results presented however do form a set of benchmark data for possible improvement and application in similar cases involving flow transportation inside pipes

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